

Mathematica 11.3 Integration Test Results

Test results for the 159 problems in "1.1.1.4 (a+b x)^m (c+d x)^n (e+x)^p (g+h x)^q.m"

Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + B x}{\sqrt{a + b x} \sqrt{c + \frac{b (-1+c) x}{a}}} \sqrt{e + \frac{b (-1+e) x}{a}} dx$$

Optimal (type 4, 145 leaves, 3 steps):

$$-\frac{2 a^{3/2} B \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c} \sqrt{a+b x}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b^2 \sqrt{1-c} (1-e)} +$$
$$\frac{2 \sqrt{a} (a B e + A (b - b e)) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c} \sqrt{a+b x}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b^2 \sqrt{1-c} (1-e)}$$

Result (type 4, 309 leaves):

$$\begin{aligned}
 & - \left(\left(2 \sqrt{\frac{a}{-1+c}} (a+b x)^{3/2} \right. \right. \\
 & \left. \left. - B \sqrt{\frac{a}{-1+c}} \left(-1+c + \frac{a}{a+b x} \right) \left(-1+e + \frac{a}{a+b x} \right) - \frac{1}{\sqrt{a+b x}} i a B (-1+e) \sqrt{\frac{-1+c + \frac{a}{a+b x}}{-1+c}} \right. \right. \\
 & \left. \left. \sqrt{\frac{-1+e + \frac{a}{a+b x}}{-1+e}} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+c}}}{\sqrt{a+b x}}\right], \frac{-1+c}{-1+e}\right] + \frac{1}{\sqrt{a+b x}} i (a B c + A (b - b c)) \right. \right. \\
 & \left. \left. (-1+e) \sqrt{\frac{-1+c + \frac{a}{a+b x}}{-1+c}} \sqrt{\frac{-1+e + \frac{a}{a+b x}}{-1+e}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+c}}}{\sqrt{a+b x}}\right], \frac{-1+c}{-1+e}\right] \right) \right) / \\
 & \left(a b^2 (-1+e) \sqrt{c + \frac{b (-1+c) x}{a}} \sqrt{e + \frac{b (-1+e) x}{a}} \right)
 \end{aligned}$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x}{\sqrt{a+b x} \sqrt{c+d x} \sqrt{e + \frac{b (-1+e) x}{a}}} dx$$

Optimal (type 4, 221 leaves, 5 steps):

$$\begin{aligned}
 & - \left(\left(2 a B \sqrt{-b c + a d} \sqrt{\frac{b (c+d x)}{b c - a d}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], - \frac{(b c - a d) (1-e)}{a d}\right] \right) \right. \\
 & \left. \left(b^2 \sqrt{d} (1-e) \sqrt{c+d x} \right) + \left(2 \sqrt{a} (a B e + A (b - b e)) \sqrt{\frac{b (c+d x)}{b c - a d}} \right. \right. \\
 & \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-e} \sqrt{a+b x}}{\sqrt{a}}\right], - \frac{a d}{(b c - a d) (1-e)}\right] \right) \right) / \left(b^2 (1-e)^{3/2} \sqrt{c+d x} \right)
 \end{aligned}$$

Result (type 4, 312 leaves):

$$\begin{aligned}
 & - \left(\left(2 \sqrt{\frac{a}{-1+e}} (a+b x)^{3/2} \right. \right. \\
 & \left. \left. - \frac{b B \sqrt{\frac{a}{-1+e}} (c+d x) (a e + b (-1+e) x)}{(a+b x)^2} - \frac{1}{\sqrt{a+b x}} \right. \right. \dot{a} B d \sqrt{\frac{b (c+d x)}{d (a+b x)}} \sqrt{\frac{-1+e+\frac{a}{a+b x}}{-1+e}} \\
 & \left. \left. \text{EllipticE}\left[\dot{x} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+b x}}\right], \frac{(b c - a d) (-1+e)}{a d}\right] + \frac{1}{\sqrt{a+b x}} \dot{x} d (a B e + A (b - b e)) \right. \right. \\
 & \left. \left. \sqrt{\frac{b (c+d x)}{d (a+b x)}} \sqrt{\frac{-1+e+\frac{a}{a+b x}}{-1+e}} \text{EllipticF}\left[\dot{x} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+b x}}\right], \frac{(b c - a d) (-1+e)}{a d}\right] \right) \right) \\
 & \left(a b^2 d \sqrt{c+d x} \sqrt{e + \frac{b (-1+e) x}{a}} \right)
 \end{aligned}$$

Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{a+b x} dx$$

Optimal (type 4, 570 leaves, 12 steps):

$$\begin{aligned}
 & \frac{2 \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 b} - \\
 & \left(2 \sqrt{-d e + c f} (3 a d f h - b (d f g + d e h + c f h)) \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{g+h x} \right. \\
 & \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) / \left(3 b^2 d \sqrt{f} h \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{d g - c h}} \right) + \\
 & \left(2 \sqrt{-d e + c f} (3 a^2 d f h^2 - 3 a b (d e + c f) h^2 - b^2 (d g (f g - e h) - c h (f g + 2 e h))) \right. \\
 & \left. \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) / \\
 & \left(3 b^3 d \sqrt{f} h \sqrt{e+f x} \sqrt{g+h x} \right) - \\
 & \left(2 (b e - a f) \sqrt{-d e + c f} (b g - a h) \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \text{EllipticPi}[\right. \\
 & \left. - \frac{b (d e - c f)}{(b c - a d) f}, \text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) / \left(b^3 \sqrt{f} \sqrt{e+f x} \sqrt{g+h x} \right)
 \end{aligned}$$

Result (type 4, 29892 leaves): Display of huge result suppressed!

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x} \sqrt{1+4x}} dx$$

Optimal (type 4, 47 leaves, 2 steps):

$$\frac{\sqrt{\frac{11}{2}} \sqrt{5-2x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{1+4x}}{\sqrt{11}}\right], 3]}{2 \sqrt{-5+2x}}$$

Result (type 4, 111 leaves):

$$- \left(\left(\frac{2 (-5 + 2 x) (-2 + 3 x)}{\sqrt{\frac{1}{2} + 2 x}} + \sqrt{11} \sqrt{\frac{-5 + 2 x}{1 + 4 x}} \sqrt{\frac{-2 + 3 x}{1 + 4 x}} \right. \right. \\
 \left. \left. (1 + 4 x) \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{11}{3}}}{\sqrt{1 + 4 x}}\right], 3] \right) \Big/ \left(2 \sqrt{2 - 3 x} \sqrt{-10 + 4 x} \right) \right)$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c + d x}}{(a + b x) \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 293 leaves, 8 steps) :

$$\left(2 \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) \Big/ \\
 \left(b \sqrt{f} \sqrt{e + f x} \sqrt{g + h x} \right) - \left(2 \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \right. \\
 \left. \text{EllipticPi}\left[-\frac{b (d e - c f)}{(b c - a d) f}, \text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right] \right) \Big/ \left(b \sqrt{f} \sqrt{e + f x} \sqrt{g + h x} \right)$$

Result (type 4, 202 leaves) :

$$- \left(\left(2 \frac{i}{2} \sqrt{c + d x} \sqrt{\frac{d (g + h x)}{d g - c h}} \right. \right. \\
 \left. \left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{f (c + d x)}{d e - c f}}\right], \frac{d e h - c f h}{d f g - c f h}\right] - \text{EllipticPi}\left[\frac{b (-d e + c f)}{(b c - a d) f}, \right. \right. \right. \\
 \left. \left. \left. i \text{ArcSinh}\left[\sqrt{\frac{f (c + d x)}{d e - c f}}\right], \frac{d e h - c f h}{d f g - c f h}\right]\right) \Big/ \left(b \sqrt{\frac{f (c + d x)}{d (e + f x)}} \sqrt{e + f x} \sqrt{g + h x} \right) \right)$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c + d x)^{3/2}}{(a + b x) \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 449 leaves, 11 steps):

$$\begin{aligned} & \left(2 d \sqrt{-f g + e h} \sqrt{c + d x} \sqrt{\frac{f (g + h x)}{f g - e h}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{h} \sqrt{e + f x}}{\sqrt{-f g + e h}}\right], -\frac{d (f g - e h)}{(d e - c f) h}\right] \right) / \\ & \left(b f \sqrt{h} \sqrt{-\frac{f (c + d x)}{d e - c f} \sqrt{g + h x}} \right) + \left(2 (b c - a d) \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right] \right) / \left(b^2 \sqrt{f} \sqrt{e + f x} \sqrt{g + h x} \right) - \\ & \left(2 (b c - a d) \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \operatorname{EllipticPi}\left[-\frac{b (d e - c f)}{(b c - a d) f}, \right. \right. \\ & \left. \left. \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right] \right) / \left(b^2 \sqrt{f} \sqrt{e + f x} \sqrt{g + h x} \right) \end{aligned}$$

Result (type 4, 381 leaves):

$$\begin{aligned}
& - \frac{1}{b^2 f h \sqrt{g + h x}} \\
& 2 \sqrt{c + d x} \left(- \frac{b d f (g + h x)}{\sqrt{e + f x}} + \left(\pm \sqrt{\frac{f (g + h x)}{h (e + f x)}} \right) \right. \\
& \left. \left(- b d^2 (b e - a f) (-f g + e h) \text{EllipticE}[\pm \text{ArcSinh}[\right. \right. \\
& \left. \left. \frac{\sqrt{-e + \frac{f g}{h}}}{\sqrt{e + f x}} \right], \frac{(d e - c f) h}{d (-f g + e h)} \right] + f \left[-b (a d^2 (-f g + e h) + b (d^2 e g - 2 c d e h + c^2 f h)) \right. \\
& \left. \left. \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-e + \frac{f g}{h}}}{\sqrt{e + f x}}, \frac{(d e - c f) h}{d (-f g + e h)} \right] + (b c - a d)^2 f h \right. \\
& \left. \left. \text{EllipticPi}[\frac{(b e - a f) h}{b (-f g + e h)}, \pm \text{ArcSinh}[\frac{\sqrt{-e + \frac{f g}{h}}}{\sqrt{e + f x}}, \frac{(d e - c f) h}{d (-f g + e h)} \right] \right] \right) \right) / \\
& \left(d (-b e + a f) \sqrt{-e + \frac{f g}{h}} \sqrt{\frac{f (c + d x)}{d (e + f x)}} \right)
\end{aligned}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c i + d i x}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 137 leaves, 3 steps):

$$\begin{aligned}
& \left(2 \sqrt{-f g + e h} i \sqrt{c + d x} \sqrt{\frac{f (g + h x)}{f g - e h}} \text{EllipticE}[\text{ArcSin}[\frac{\sqrt{h} \sqrt{e + f x}}{\sqrt{-f g + e h}}, -\frac{d (f g - e h)}{(d e - c f) h}] \right) / \\
& \left(f \sqrt{h} \sqrt{-\frac{f (c + d x)}{d e - c f}} \sqrt{g + h x} \right)
\end{aligned}$$

Result (type 4, 180 leaves):

$$-\left(\left(2 \pm i \sqrt{c+d x} \sqrt{g+h x} \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{f(c+d x)}{d e - c f}} \right], \frac{d e h - c f h}{d f g - c f h} \right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{f(c+d x)}{d e - c f}} \right], \frac{d e h - c f h}{d f g - c f h} \right] \right) \right) / \left(h \sqrt{\frac{f(c+d x)}{d(e+f x)}} \sqrt{e+f x} \sqrt{\frac{d(g+h x)}{d g - c h}} \right) \right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 284 leaves, 6 steps):

$$\begin{aligned} & \left(2 b \sqrt{-d e + c f} \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{g+h x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}} \right], \frac{(d e - c f) h}{f(d g - c h)} \right] \right) / \\ & \left(d \sqrt{f} h \sqrt{e+f x} \sqrt{\frac{d(g+h x)}{d g - c h}} \right) - \left(2 \sqrt{-d e + c f} (b g - a h) \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{\frac{d(g+h x)}{d g - c h}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}} \right], \frac{(d e - c f) h}{f(d g - c h)} \right] \right) / \left(d \sqrt{f} h \sqrt{e+f x} \sqrt{g+h x} \right) \end{aligned}$$

Result (type 4, 319 leaves):

$$\begin{aligned} & - \left(\left(2 \left(-b d^2 \sqrt{-c + \frac{d e}{f}} (e+f x) (g+h x) - \pm b (d e - c f) h (c+d x)^{3/2} \sqrt{\frac{d(e+f x)}{f(c+d x)}} \sqrt{\frac{d(g+h x)}{h(c+d x)}} \right. \right. \right. \\ & \left. \left. \left. \text{EllipticE}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c+d x}} \right], \frac{d f g - c f h}{d e h - c f h} \right] + \pm d (b e - a f) h (c+d x)^{3/2} \right. \right) \\ & \left. \left. \left. \sqrt{\frac{d(e+f x)}{f(c+d x)}} \sqrt{\frac{d(g+h x)}{h(c+d x)}} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c+d x}} \right], \frac{d f g - c f h}{d e h - c f h} \right] \right) \right) / \\ & \left(d^2 \sqrt{-c + \frac{d e}{f}} f h \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \right) \right) \end{aligned}$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 165 leaves, 4 steps):

$$-\left(\left(2 \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \text{EllipticPi}\left[-\frac{b (d e - c f)}{(b c - a d) f}, \right. \right. \right. \\ \left. \left. \left. \text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right] \right) / \left((b c - a d) \sqrt{f} \sqrt{e + f x} \sqrt{g + h x} \right)$$

Result (type 4, 225 leaves):

$$\left(2 \text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right] \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \left(\text{EllipticF}\left[\text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] - \right. \right. \\ \left. \left. \text{EllipticPi}\left[\frac{(b c - a d) f}{b (-d e + c f)}, \text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right]\right) \right) / \left((-b c + a d) \sqrt{-c + \frac{d e}{f}} \sqrt{e + f x} \sqrt{g + h x} \right)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + b x) (c + d x)^{3/2} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 393 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 d^2 \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (d e - c f) (d g - c h) \sqrt{c + d x}} - \\
& \left(2 d \sqrt{h} \sqrt{-f g + e h} \sqrt{c + d x} \sqrt{\frac{f (g + h x)}{f g - e h}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{h} \sqrt{e + f x}}{\sqrt{-f g + e h}}\right], -\frac{d (f g - e h)}{(d e - c f) h}\right] \right) / \\
& \left((b c - a d) (d e - c f) (d g - c h) \sqrt{-\frac{f (c + d x)}{d e - c f}} \sqrt{g + h x} \right) - \\
& \left(2 b \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \text{EllipticPi}\left[-\frac{b (d e - c f)}{(b c - a d) f}, \right. \right. \\
& \left. \left. \text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right] \right) / ((b c - a d)^2 \sqrt{f} \sqrt{e + f x} \sqrt{g + h x})
\end{aligned}$$

Result (type 4, 321 leaves):

$$\begin{aligned}
& \left(2 \text{Im} (c + d x) \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \right. \\
& \left((b c - a d) f \text{EllipticE}\left[\text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d g}{h}}}{\sqrt{c + d x}}\right], \frac{d e h - c f h}{d f g - c f h}\right] + \right. \\
& (b d e - 2 b c f + a d f) \text{EllipticF}\left[\text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d g}{h}}}{\sqrt{c + d x}}\right], \frac{d e h - c f h}{d f g - c f h}\right] + \\
& \left. b (-d e + c f) \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \text{ArcSinh}\left[\frac{\sqrt{-c + \frac{d g}{h}}}{\sqrt{c + d x}}\right], \frac{d e h - c f h}{d f g - c f h}\right]\right) / \\
& \left((b c - a d)^2 (-d e + c f) \sqrt{-c + \frac{d g}{h}} \sqrt{e + f x} \sqrt{g + h x}\right)
\end{aligned}$$

Problem 72: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) (c + d x)^{5/2} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 875 leaves, 18 steps):

$$\begin{aligned}
& \frac{2 d^2 \sqrt{e+f x} \sqrt{g+h x}}{3 (b c-a d) (d e-c f) (d g-c h) (c+d x)^{3/2}} + \\
& \frac{2 b d^2 \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (d e-c f) (d g-c h) \sqrt{c+d x}} - \frac{4 d^2 (d f g+d e h-2 c f h) \sqrt{e+f x} \sqrt{g+h x}}{3 (b c-a d) (d e-c f)^2 (d g-c h)^2 \sqrt{c+d x}} + \\
& \left(4 d \sqrt{f} (d f g+d e h-2 c f h) \sqrt{\frac{d (e+f x)}{d e-c f}} \sqrt{g+h x} \right. \\
& \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}] \right) / \\
& \left(3 (b c-a d) (-d e+c f)^{3/2} (d g-c h)^2 \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{d g-c h}} \right) - \left(2 b d \sqrt{h} \sqrt{-f g+e h} \right. \\
& \left. \sqrt{c+d x} \sqrt{\frac{f (g+h x)}{f g-e h}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{h} \sqrt{e+f x}}{\sqrt{-f g+e h}}\right], -\frac{d (f g-e h)}{(d e-c f) h}] \right) / \\
& \left((b c-a d)^2 (d e-c f) (d g-c h) \sqrt{-\frac{f (c+d x)}{d e-c f}} \sqrt{g+h x} \right) - \\
& \left(2 \sqrt{f} (2 d f g+d e h-3 c f h) \sqrt{\frac{d (e+f x)}{d e-c f}} \sqrt{\frac{d (g+h x)}{d g-c h}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}] \right) / \\
& \left(3 (b c-a d) (-d e+c f)^{3/2} (d g-c h) \sqrt{e+f x} \sqrt{g+h x} \right) - \\
& \left(2 b^2 \sqrt{-d e+c f} \sqrt{\frac{d (e+f x)}{d e-c f}} \sqrt{\frac{d (g+h x)}{d g-c h}} \text{EllipticPi}\left[-\frac{b (d e-c f)}{(b c-a d) f}, \right. \right. \\
& \left. \left. \text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right] \right) / \left((b c-a d)^3 \sqrt{f} \sqrt{e+f x} \sqrt{g+h x} \right)
\end{aligned}$$

Result(type 4, 12191 leaves):

$$\begin{aligned}
& \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \left(\frac{2 d^2}{3 (b c-a d) (-d e+c f) (-d g+c h) (c+d x)^2} + \right. \\
& \left. (2 d^2 (3 b d^2 e g - 5 b c d f g + 2 a d^2 f g - 5 b c d e h + 2 a d^2 e h + 7 b c^2 f h - 4 a c d f h)) \right) /
\end{aligned}$$

$$\begin{aligned}
 & \left(3 (b c - a d)^2 (-d e + c f)^2 (-d g + c h)^2 (c + d x) \right) \Bigg) + \frac{1}{3 (-b c + a d)^2 (-d e + c f)^2 (-d g + c h)^2} \\
 & 2 \left(\left(-3 b d^2 e g + 5 b c d f g - 2 a d^2 f g + 5 b c d e h - 2 a d^2 e h - 7 b c^2 f h + 4 a c d f h \right) \right. \\
 & \quad \left(c + d x \right)^{3/2} \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) \Bigg) / \\
 & \quad \left(\sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c + d x} \right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c + d x} \right)}{d}} \right) + \\
 & \quad \left(c + d x \right) \left(-b + \frac{b c}{c + d x} - \frac{a d}{c + d x} \right) \sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}} \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}} \\
 & \quad \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right)} \\
 & \quad \left((b c - a d) (-d e + c f) h (-b d^2 f g^2 + b d^2 e g h + a d^2 f g h - 3 b c d e h^2 + 2 a d^2 e h^2 + 3 b c^2 \right. \\
 & \quad \left. f h^2 - 3 a c d f h^2) \right) / \left(d (b g - a h) \sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}} \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}} \right) + \\
 & \quad \frac{3 b d^3 e^2 g \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}}}{\sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}}} - \frac{8 b c d^2 e f g \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}}}{\sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}}} + \\
 & \quad \frac{2 a d^3 e f g \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}}}{\sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}}} + \frac{5 b c^2 d f^2 g \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}}}{\sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}}} -
 \end{aligned}$$

$$\begin{aligned}
& \frac{2 a c d^2 f^2 g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{5 b c d^2 e^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \\
& \frac{2 a d^3 e^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \frac{12 b c^2 d e f h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \\
& \frac{6 a c d^2 e f h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{7 b c^3 f^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \\
& \frac{4 a c^2 d f^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \left. \frac{3 b^3 (d e - c f)^2 (d g - c h)^2 \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{d (b g - a h) \left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x} \right) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} \right) \\
& \left(\left(3 \pm b d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right) \left(\text{EllipticE} \left[\pm \text{ArcSinh} \left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right] \right], \right. \right. \\
& \left. \left. \frac{(-d e + c f) h}{f (-d g + c h)} \right) - \text{EllipticF} \left[\pm \text{ArcSinh} \left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) / \\
& \left((b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \right. \\
& \left. \left. \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \left(5 \pm b c d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)
\right)
\end{aligned}$$

$$\begin{aligned}
 & \left(\text{EllipticE} \left[\text{i ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[\right. \right. \\
 & \quad \left. \left. \text{i ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) / \left((b c - a d) (-d e + c f) \right. \\
 & \quad \left. \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x} \right)} \right) + \\
 & \quad \left(2 i a d^2 f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left(\text{EllipticE} \left[\text{i ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[\text{i ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
 & \quad \left((b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{d f g + d e h - 2 c f h}{c + d x} \right) \right) - \left(5 i b c d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
 & \quad \left(\text{EllipticE} \left[\text{i ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[\right. \right. \\
 & \quad \left. \left. \text{i ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) / \left((b c - a d) (-d e + c f) \right)
 \end{aligned}$$

$$\begin{aligned}
& \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x} \right)} + \\
& \left(2 \pm a d^2 e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left(\begin{array}{l} \text{EllipticE}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \right. \\ \left. \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \end{array} \right) \right) / \\
& \left((b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \right.} \right. \\
& \left. \left. \frac{d f g + d e h - 2 c f h}{c + d x} \right) + \left(7 \pm b c^2 f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \right. \\
& \left. \left. \left(\begin{array}{l} \text{EllipticE}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \right. \\ \left. \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. \pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right) \right) / \left((b c - a d) (-d e + c f) \right. \\
& \left. \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x} \right)} \right) - \\
& \left(4 \pm a c d f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left(\begin{array}{l} \text{EllipticE}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \right. \\ \left. \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. \pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \left. \left(\frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[\text{i} \operatorname{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \Bigg) \Bigg) \\
 & \left((b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \frac{d f g + d e h - 2 c f h}{c + d x} \right) \right) - \left(3 \pm b^2 d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \right. \\
 & \left. \left. \left. \left. \text{EllipticF} \left[\text{i} \operatorname{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) \Bigg) \Bigg) \Bigg) \\
 & \left. \left. \left. \left. \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) + \right. \right. \right. \right. \\
 & \left. \left. \left. \left. 5 \pm b^2 c d f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[\right. \right. \right. \right. \\
 & \left. \left. \left. \left. \text{i} \operatorname{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) \\
 & \left. \left. \left. \left. \sqrt{\left(f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \right. \right. \right. \right. \\
 & \left. \left. \left. \left. 2 \pm a b d^2 f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[\right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& \left. \frac{\text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}}{\left(b c-a d\right)^2 \sqrt{-\frac{-d g+c h}{h}}} \right\} / \\
& \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)} - \\
& \left. \left\{ 2 \pm b d f g \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \right.\right. \right. \\
& \left. \left. \left. \frac{(-d e+c f) h}{f (-d g+c h)}\right]\right\} / \left. \left\{ \left(b c-a d\right) \sqrt{-\frac{-d g+c h}{h}} \right. \right. \\
& \left. \left. \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)} + \right. \right. \\
& \left. \left. \left. \left\{ 5 \pm b^2 c d e h \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}\right]\right\} / \left. \left. \left. \left\{ \left(b c-a d\right)^2 \sqrt{-\frac{-d g+c h}{h}} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)} - \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 2 \pm a b d^2 e h \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{\text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}}{\left(b c-a d\right)^2 \sqrt{-\frac{-d g+c h}{h}}} \right\} / \\
 & \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)} - \\
 & \left. \left(2 \pm b d e h \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \right.\right.\right. \\
 & \left.\left.\left.\frac{(-d e+c f) h}{f (-d g+c h)}\right]\right\} / \left(\left(b c-a d\right) \sqrt{-\frac{-d g+c h}{h}}\right. \\
 & \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)} - \\
 & \left. \left(7 \pm b^2 c^2 f h \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\right.\right.\right. \\
 & \left.\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}\right]\right\} / \left(\left(b c-a d\right)^2 \sqrt{-\frac{-d g+c h}{h}}\right. \\
 & \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)} + \\
 & \left. \left(4 \pm a b c d f h \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\right.\right.\right.
 \end{aligned}$$

$$\begin{aligned}
& \left. \frac{\text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}}{\left(b c-a d\right)^2 \sqrt{-\frac{-d g+c h}{h}}} \right\} \\
& \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)}+ \\
& \left. \left\{5 \pm b c f h \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \right.\right. \right. \\
& \left. \left. \left. \frac{(-d e+c f) h}{f (-d g+c h)}\right]\right\} \middle/ \left. \left(\left(b c-a d\right) \sqrt{-\frac{-d g+c h}{h}}\right)\right. \\
& \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)}- \\
& \left. \left\{\pm a d f h \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \right.\right. \right. \\
& \left. \left. \left. \frac{(-d e+c f) h}{f (-d g+c h)}\right]\right\} \middle/ \left. \left(\left(b c-a d\right) \sqrt{-\frac{-d g+c h}{h}}\right)\right. \\
& \sqrt{\left(f h+\frac{d^2 e g-c d f g-c d e h+c^2 f h}{(c+d x)^2}+\frac{d f g+d e h-2 c f h}{c+d x}\right)}+\frac{1}{\left(b c-a d\right)^3} \\
& 3 b^3 d^2 e g \left(\left\{ \pm c \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c-a d) h}{b (-d g+c h)}, \right.\right. \right. \\
& \left. \left. \left. \right.\right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{\text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}}{\sqrt{-\frac{-d g+c h}{h}}} \sqrt{f h +} \right\} - \\
 & \left. \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x} \right\} - \\
 & \left. \left(\frac{\text{ad} \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c-a d) h}{b (-d g+c h)}, \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}\right]}{b \sqrt{-\frac{-d g+c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2}} - \right. \right. \\
 & \left. \left. \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x} \right) \right) - \\
 & \frac{1}{(b c-a d)^3} 5 b^3 c d f g \left(\left(\frac{\text{ad} \sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c-a d) h}{b (-d g+c h)}, \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}\right]}{b \sqrt{-\frac{-d g+c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2}} - \right. \right. \\
 & \left. \left. \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x} \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
& \left. \frac{\text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}}} \right\} \\
& \left. \left(b \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right)} \right) \right\} + \frac{1}{(b c - a d)^3} \\
& 2 a b^2 d^2 f g \left(\left. \left(i c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right) \right\} \right. \\
& \left. \left. \left. \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right)} \right) \right\} - \right. \\
& \left. \left. \left. i a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right) \right\} \right. \\
& \left. \left. \left. \left. b \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right)} \right) \right) \right\} + \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{1}{(b c - a d)^2} 2 b^2 d f g \left(\left(\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi} \right. \right. \\
 & \left. \left. \frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh} \left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right) \right) / \\
 & \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \right. \right. \right. \\
 & \left. \left. \left. \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) - \left(\pm a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
 & \left. \left. \left. \text{EllipticPi} \left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh} \left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
 & \left(b \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \right. \right. \right. \\
 & \left. \left. \left. \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) \right) - \frac{1}{(b c - a d)^3} \\
 & 5 b^3 c d e h \left(\left(\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi} \left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \right. \\
 & \left. \left. \left. \pm \text{ArcSinh} \left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \right. \right. \right. \\
 & \left. \left. \left. \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) - \right. \\
& \left(\pm a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right. / \left(b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2}} - \right. \\
& \left. \left. \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) + \\
& \frac{1}{(b c - a d)^3} 2 a b^2 d^2 e h \left(\left(\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right) / \right. \\
& \left. \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2}} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \right. \right. \\
& \left. \left. \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) - \left(\pm a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right)
\end{aligned}$$

$$\begin{aligned}
 & \left(b \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \right.} \right. \\
 & \left. \left. \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) + \frac{1}{(b c - a d)^2} \\
 & 2 b^2 d e h \left(\text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \\
 & \left. \left. \text{i} c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right] \right. \\
 & \left. \left. \text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) / \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \right.} \right. \\
 & \left. \left. \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) - \\
 & \left(\text{i} a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \\
 & \left. \left. \text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) / \left(b \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \right.} \right. \\
 & \left. \left. \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) + \\
 & \frac{1}{(b c - a d)^3} 7 b^3 c^2 f h \left(\text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \\
 & \left. \left. \text{i} c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right]
 \end{aligned}$$

$$\begin{aligned}
& \left. \frac{(b c - a d) h}{b (-d g + c h)}, \text{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right\} / \\
& \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \right.} \right. \\
& \left. \left. \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) - \left(\pm a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \text{EllipticPi} \left[\frac{(b c - a d) h}{b (-d g + c h)}, \text{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right\} / \right. \\
& \left. \left. b \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \right.} \right. \right. \\
& \left. \left. \left. \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) \right) - \frac{1}{(b c - a d)^3} \\
& 4 a b^2 c d f h \left(\left(\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi} \left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \right. \\
& \left. \left. \left. \text{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right\} / \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \right.} \right. \\
& \left. \left. \left. \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) - \right)
\end{aligned}$$

$$\begin{aligned}
 & \left(\frac{\text{a} \text{d}}{\sqrt{1 - \frac{-\text{d} \text{e} + \text{c} \text{f}}{\text{f} (\text{c} + \text{d} x)}}} \sqrt{1 - \frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h} (\text{c} + \text{d} x)}} \operatorname{EllipticPi}\left[\frac{(\text{b} \text{c} - \text{a} \text{d}) \text{h}}{\text{b} (-\text{d} \text{g} + \text{c} \text{h})}, \text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h}}}}{\sqrt{\text{c} + \text{d} x}}, \frac{(-\text{d} \text{e} + \text{c} \text{f}) \text{h}}{\text{f} (-\text{d} \text{g} + \text{c} \text{h})}\right]\right] \right) / \\
 & \left(\text{b} \sqrt{-\frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h}}} \sqrt{\left(\text{f} \text{h} + \frac{\text{d}^2 \text{e} \text{g}}{(\text{c} + \text{d} x)^2} - \frac{\text{c} \text{d} \text{f} \text{g}}{(\text{c} + \text{d} x)^2} - \frac{\text{c} \text{d} \text{e} \text{h}}{(\text{c} + \text{d} x)^2} + \frac{\text{c}^2 \text{f} \text{h}}{(\text{c} + \text{d} x)^2} + \frac{\text{d} \text{f} \text{g}}{\text{c} + \text{d} x} + \frac{\text{d} \text{e} \text{h}}{\text{c} + \text{d} x} - \frac{2 \text{c} \text{f} \text{h}}{\text{c} + \text{d} x}\right)}\right) - \\
 & \frac{1}{(\text{b} \text{c} - \text{a} \text{d})^2} 5 \text{b}^2 \text{c} \text{f} \text{h} \left(\left(\text{i} \text{c} \sqrt{1 - \frac{-\text{d} \text{e} + \text{c} \text{f}}{\text{f} (\text{c} + \text{d} x)}} \sqrt{1 - \frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h} (\text{c} + \text{d} x)}} \operatorname{EllipticPi}\left[\frac{(\text{b} \text{c} - \text{a} \text{d}) \text{h}}{\text{b} (-\text{d} \text{g} + \text{c} \text{h})}, \text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h}}}}{\sqrt{\text{c} + \text{d} x}}, \frac{(-\text{d} \text{e} + \text{c} \text{f}) \text{h}}{\text{f} (-\text{d} \text{g} + \text{c} \text{h})}\right]\right] \right) / \\
 & \left(\sqrt{-\frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h}}} \sqrt{\left(\text{f} \text{h} + \frac{\text{d}^2 \text{e} \text{g}}{(\text{c} + \text{d} x)^2} - \frac{\text{c} \text{d} \text{f} \text{g}}{(\text{c} + \text{d} x)^2} - \frac{\text{c} \text{d} \text{e} \text{h}}{(\text{c} + \text{d} x)^2} + \frac{\text{c}^2 \text{f} \text{h}}{(\text{c} + \text{d} x)^2} + \frac{\text{d} \text{f} \text{g}}{\text{c} + \text{d} x} + \frac{\text{d} \text{e} \text{h}}{\text{c} + \text{d} x} - \frac{2 \text{c} \text{f} \text{h}}{\text{c} + \text{d} x}\right)}\right) - \\
 & \left(\text{i} \text{a} \text{d} \sqrt{1 - \frac{-\text{d} \text{e} + \text{c} \text{f}}{\text{f} (\text{c} + \text{d} x)}} \sqrt{1 - \frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h} (\text{c} + \text{d} x)}} \operatorname{EllipticPi}\left[\frac{(\text{b} \text{c} - \text{a} \text{d}) \text{h}}{\text{b} (-\text{d} \text{g} + \text{c} \text{h})}, \text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h}}}}{\sqrt{\text{c} + \text{d} x}}, \frac{(-\text{d} \text{e} + \text{c} \text{f}) \text{h}}{\text{f} (-\text{d} \text{g} + \text{c} \text{h})}\right]\right] \right) / \\
 & \left(\text{b} \sqrt{-\frac{-\text{d} \text{g} + \text{c} \text{h}}{\text{h}}} \sqrt{\left(\text{f} \text{h} + \frac{\text{d}^2 \text{e} \text{g}}{(\text{c} + \text{d} x)^2} - \frac{\text{c} \text{d} \text{f} \text{g}}{(\text{c} + \text{d} x)^2} - \frac{\text{c} \text{d} \text{e} \text{h}}{(\text{c} + \text{d} x)^2} + \frac{\text{c}^2 \text{f} \text{h}}{(\text{c} + \text{d} x)^2} + \frac{\text{d} \text{f} \text{g}}{\text{c} + \text{d} x} + \frac{\text{d} \text{e} \text{h}}{\text{c} + \text{d} x} - \frac{2 \text{c} \text{f} \text{h}}{\text{c} + \text{d} x}\right)}\right)
 \end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) + \frac{1}{(b c - a d)^2} \\
& a b d f h \left(\left(\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi} \left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \right. \\
& \left. \left. \left. \pm \text{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right] \right) \Big/ \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \right. \right. \\
& \left. \left. \left. \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) - \\
& \left(\pm a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi} \left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \right. \\
& \left. \left. \left. \pm \text{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right] \right) \Big/ \left(b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \right. \right. \\
& \left. \left. \left. \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) \right) + \frac{1}{b c - a d} \\
& b f h \left(\left(\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi} \left[\frac{(b c - a d) h}{b (-d g + c h)}, \right. \right. \right. \\
& \left. \left. \left. \pm \text{ArcSinh} \left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right] \right) \Big/ \left(\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \right. \right. \\
& \left. \left. \left. \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x} \right) \right) -
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x} \right) - \\
 & \left(\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\right. \right. \right. \\
 & \left. \left. \left. \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right] / \left(b \sqrt{-\frac{-d g + c h}{h}} \sqrt{\left(f h + \frac{d^2 e g}{(c+d x)^2} - \right.} \right. \\
 & \left. \left. \left. \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x} \right) \right) \right) \right) / \\
 & \left(\left(b f h + \frac{3 b d^2 e g}{(c+d x)^2} - \frac{5 b c d f g}{(c+d x)^2} + \frac{2 a d^2 f g}{(c+d x)^2} - \frac{5 b c d e h}{(c+d x)^2} + \frac{2 a d^2 e h}{(c+d x)^2} + \frac{7 b c^2 f h}{(c+d x)^2} - \right. \right. \\
 & \left. \left. \frac{4 a c d f h}{(c+d x)^2} + \frac{2 b d f g}{c+d x} + \frac{2 b d e h}{c+d x} - \frac{5 b c f h}{c+d x} + \frac{a d f h}{c+d x} \right) \right. \\
 & \left. \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}} \right)
 \end{aligned}$$

Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b x) \sqrt{c+d x} \sqrt{1-f x} \sqrt{1+f x}} dx$$

Optimal (type 4, 74 leaves, 3 steps):

$$\frac{2 \sqrt{\frac{f (c+d x)}{d+c f}} \text{EllipticPi}\left[\frac{2 b}{b+a f}, \text{ArcSin}\left[\frac{\sqrt{1-f x}}{\sqrt{2}}\right], \frac{2 d}{d+c f}\right]}{(b+a f) \sqrt{c+d x}}$$

Result (type 4, 203 leaves):

$$\left(\begin{array}{l} 2 \operatorname{Im}((c + d x) \sqrt{\frac{d (-1 + f x)}{f (c + d x)}} \sqrt{\frac{d + d f x}{c f + d f x}} \\ \operatorname{EllipticF}[\operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}] - \operatorname{EllipticPi}\left[\frac{b c f - a d f}{b d + b c f}, \right. \\ \left. \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}\right] \end{array} \right) \Bigg/ \left((-b c + a d) \sqrt{-\frac{d+c f}{f}} \sqrt{1 - f^2 x^2} \right)$$

Problem 74: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{1 - f^2 x^2}} dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$-\frac{2 \sqrt{\frac{f (c+d x)}{d+c f}} \operatorname{EllipticPi}\left[\frac{2 b}{b+a f}, \operatorname{ArcSin}\left[\frac{\sqrt{1-f x}}{\sqrt{2}}\right], \frac{2 d}{d+c f}\right]}{(b+a f) \sqrt{c+d x}}$$

Result (type 4, 203 leaves):

$$\left(\begin{array}{l} 2 \operatorname{Im}((c + d x) \sqrt{\frac{d (-1 + f x)}{f (c + d x)}} \sqrt{\frac{d + d f x}{c f + d f x}} \\ \operatorname{EllipticF}[\operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}] - \operatorname{EllipticPi}\left[\frac{b c f - a d f}{b d + b c f}, \right. \\ \left. \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}\right] \end{array} \right) \Bigg/ \left((-b c + a d) \sqrt{-\frac{d+c f}{f}} \sqrt{1 - f^2 x^2} \right)$$

Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{1 - f^2 x} \sqrt{1 + f^2 x}} dx$$

Optimal (type 4, 86 leaves, 3 steps):

$$\frac{2 \sqrt{\frac{f^2 (c+d x)}{d+c f^2}} \text{EllipticPi}\left[\frac{2 b}{b+a f^2}, \text{ArcSin}\left[\frac{\sqrt{1-f^2 x}}{\sqrt{2}}\right], \frac{2 d}{d+c f^2}\right]}{(b+a f^2) \sqrt{c+d x}}$$

Result (type 4, 218 leaves):

$$\begin{aligned} & \left(2 \pm (c + d x) \sqrt{\frac{d (-1 + f^2 x)}{f^2 (c + d x)}} \sqrt{\frac{d (1 + f^2 x)}{f^2 (c + d x)}} \right. \\ & \left. \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + d x}}\right], \frac{-d + c f^2}{d + c f^2}\right] - \text{EllipticPi}\left[\frac{(b c - a d) f^2}{b (d + c f^2)}, \right. \right. \\ & \left. \left. \pm \text{ArcSinh}\left[\frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + d x}}\right], \frac{-d + c f^2}{d + c f^2}\right] \right) / \left((-b c + a d) \sqrt{-c - \frac{d}{f^2}} \sqrt{1 - f^4 x^2} \right) \end{aligned}$$

Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{1 - f^4 x^2}} dx$$

Optimal (type 4, 86 leaves, 4 steps):

$$\frac{2 \sqrt{\frac{f^2 (c+d x)}{d+c f^2}} \text{EllipticPi}\left[\frac{2 b}{b+a f^2}, \text{ArcSin}\left[\frac{\sqrt{1-f^2 x}}{\sqrt{2}}\right], \frac{2 d}{d+c f^2}\right]}{(b+a f^2) \sqrt{c+d x}}$$

Result (type 4, 218 leaves):

$$\left(\begin{array}{l} 2 \operatorname{Int} (c + d x) \sqrt{\frac{d (-1 + f^2 x)}{f^2 (c + d x)}} \sqrt{\frac{d (1 + f^2 x)}{f^2 (c + d x)}} \\ \operatorname{EllipticF}[\operatorname{ArcSinh}\left[\frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + d x}}\right], \frac{-d + c f^2}{d + c f^2}] - \operatorname{EllipticPi}\left[\frac{(b c - a d) f^2}{b (d + c f^2)}, \right. \\ \left. \operatorname{ArcSinh}\left[\frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + d x}}\right], \frac{-d + c f^2}{d + c f^2}\right] \end{array} \right) / \left((-b c + a d) \sqrt{-c - \frac{d}{f^2}} \sqrt{1 - f^4 x^2} \right)$$

Problem 97: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 - 3 x}}{\sqrt{-5 + 2 x} \sqrt{1 + 4 x} (7 + 5 x)^{3/2}} dx$$

Optimal (type 4, 60 leaves, 5 steps) :

$$\frac{2 \sqrt{\frac{11}{39}} \sqrt{5 - 2 x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{39}{22}} \sqrt{1+4 x}}{\sqrt{7+5 x}}\right], \frac{62}{39}\right]}{23 \sqrt{-5 + 2 x}}$$

Result (type 4, 237 leaves) :

$$\begin{aligned} & \left(\sqrt{-5 + 2 x} \sqrt{1 + 4 x} \right. \\ & \left(-1922 \sqrt{\frac{7 + 5 x}{-2 + 3 x}} (-5 - 18 x + 8 x^2) + 62 \sqrt{682} \sqrt{\frac{-5 - 18 x + 8 x^2}{(2 - 3 x)^2}} (-14 + 11 x + 15 x^2) \right. \\ & \quad \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{31}{39}} \sqrt{\frac{-5 + 2 x}{-2 + 3 x}}\right], \frac{39}{62}\right] - 23 \sqrt{682} \sqrt{\frac{-5 - 18 x + 8 x^2}{(2 - 3 x)^2}} \\ & \quad \left. \left. (-14 + 11 x + 15 x^2) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{31}{39}} \sqrt{\frac{-5 + 2 x}{-2 + 3 x}}\right], \frac{39}{62}\right]\right) \right) / \\ & \left(27807 \sqrt{2 - 3 x} \sqrt{7 + 5 x} \sqrt{\frac{7 + 5 x}{-2 + 3 x}} (-5 - 18 x + 8 x^2) \right) \end{aligned}$$

Problem 99: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x} \sqrt{c+d x}}{\sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 721 leaves, 7 steps) :

$$\begin{aligned} & \frac{\sqrt{a+b x} \sqrt{c+d x} \sqrt{g+h x}}{h \sqrt{e+f x}} - \\ & \left(\frac{(d e - c f) (g+h x)}{(d g - c h) (e+f x)} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{f g - e h} \sqrt{c+d x}}{\sqrt{d g - c h} \sqrt{e+f x}}\right], \right. \\ & \left. \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)} \right) / \left(f h \sqrt{-\frac{(d e - c f) (a+b x)}{(b c - a d) (e+f x)}} \sqrt{g+h x} \right) + \\ & \left((d e - c f) (b f g + b e h - 2 a f h) \sqrt{\frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}} \sqrt{g+h x} \right. \\ & \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{a+b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}] \right) / \\ & \left(f^2 h \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c+d x} \sqrt{-\frac{(b e - a f) (g+h x)}{(f g - e h) (a+b x)}} \right) + \\ & \left(\sqrt{b g - a h} (a d f h - b (d f g + d e h - c f h)) \sqrt{\frac{(f g - e h) (a+b x)}{(b g - a h) (e+f x)}} \sqrt{\frac{(f g - e h) (c+d x)}{(d g - c h) (e+f x)}} \right. \\ & \left. (e+f x) \text{EllipticPi}\left[\frac{f (b g - a h)}{(b e - a f) h}, \text{ArcSin}\left[\frac{\sqrt{b e - a f} \sqrt{g+h x}}{\sqrt{b g - a h} \sqrt{e+f x}}\right], \frac{(d e - c f) (b g - a h)}{(b e - a f) (d g - c h)}\right] \right) / \\ & \left(f^2 \sqrt{b e - a f} h^2 \sqrt{a+b x} \sqrt{c+d x} \right) \end{aligned}$$

Result (type 4, 6667 leaves) :

$$\begin{aligned} & -\frac{1}{f} 2 \left(- \left(\left(\sqrt{e+f x} \left(h + \frac{f g}{e+f x} - \frac{e h}{e+f x} \right) \sqrt{a + \frac{(e+f x) \left(b - \frac{b e}{e+f x} \right)}{f}} \sqrt{c + \frac{(e+f x) \left(d - \frac{d e}{e+f x} \right)}{f}} \right) \right. \right. \\ & \left. \left. \left(2 h \sqrt{g + \frac{(e+f x) \left(h - \frac{e h}{e+f x} \right)}{f}} \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \left(1 / \left(2 h^2 \sqrt{e + f x} \left(b - \frac{b e}{e + f x} + \frac{a f}{e + f x} \right) \left(d - \frac{d e}{e + f x} + \frac{c f}{e + f x} \right) \sqrt{g + \frac{(e + f x) \left(h - \frac{e h}{e + f x} \right)}{f}} \right) \right) \\
& f (b g - a h) (f g - e h) \sqrt{\left(\left(b - \frac{b e}{e + f x} + \frac{a f}{e + f x} \right) \left(d - \frac{d e}{e + f x} + \frac{c f}{e + f x} \right) \left(h + \frac{f g}{e + f x} - \frac{e h}{e + f x} \right) \right)} \\
& \sqrt{a + \frac{(e + f x) \left(b - \frac{b e}{e + f x} \right)}{f}} \sqrt{c + \frac{(e + f x) \left(d - \frac{d e}{e + f x} \right)}{f}} \\
& \left(\left(d \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e + f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \left(-\frac{h}{-f g + e h} + \frac{1}{e + f x} \right) \right. \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x} \right)}{f (-d g + c h)}} \right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}] \Bigg) \\
& \left. \left. \left(\sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x} \right) \left(d + \frac{-d e + c f}{e + f x} \right) \left(h + \frac{f g - e h}{e + f x} \right)} \right) - \right. \right. \\
& \left. \left. d e \sqrt{-\frac{(b e - a f) (-f g + e h) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x} \right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x} \right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right. \right. \\
& \left. \left. \left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x} \right)}{f (-d g + c h)}} \right], \frac{(b e - a f) (-f g + e h)}{(d e - c f) (-b g + a h)}] \right. \right. \right. \\
& \left. \left. \left. \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)} \right) \right) / ((b e - a f) (-f g + e h)) - \frac{1}{b e - a f} b \text{EllipticF}[\right. \\
& \left. \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x} \right)}{f (-d g + c h)}} \right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)} \right] \right) \right) \\
& \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x} \right) \left(d + \frac{-d e + c f}{e + f x} \right) \left(h + \frac{f g - e h}{e + f x} \right)} \right) +
\end{aligned}$$

$$\begin{aligned}
 & \left(c f \sqrt{-\frac{(b e - a f) (-f g + e h) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right. \\
 & \left. \left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right] \right) / \left((b e - a f) (-f g + e h) \right) - \frac{1}{b e - a f} b \text{EllipticF} \right. \\
 & \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}, \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right] \right) / \\
 & \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) - \\
 & \left(1 / \left(2 d h \sqrt{e + f x} \left(b - \frac{b e}{e + f x} + \frac{a f}{e + f x}\right) \left(d - \frac{d e}{e + f x} + \frac{c f}{e + f x}\right) \sqrt{g + \frac{(e + f x) \left(h - \frac{e h}{e + f x}\right)}{f}} \right) \right) \\
 & (b c - a d) f (-d e + c f) \\
 & \sqrt{\left(\left(b - \frac{b e}{e + f x} + \frac{a f}{e + f x}\right) \left(d - \frac{d e}{e + f x} + \frac{c f}{e + f x}\right) \left(h + \frac{f g}{e + f x} - \frac{e h}{e + f x}\right)\right)} \\
 & \sqrt{a + \frac{(e + f x) \left(b - \frac{b e}{e + f x}\right)}{f}} \sqrt{c + \frac{(e + f x) \left(d - \frac{d e}{e + f x}\right)}{f}} \\
 & \left(h \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e + f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \left(-\frac{h}{-f g + e h} + \frac{1}{e + f x}\right) \right. \\
 & \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}, \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right] \right) / \\
 & \left(\sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) +
 \end{aligned}$$

$$\begin{aligned}
& \left(f g \sqrt{-\frac{(b e - a f) (-f g + e h) \left(-\frac{b}{e+a f} + \frac{1}{e+f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right. \\
& \left. \left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}]\right) \middle/ \left((b e - a f) (-f g + e h) \right) - \frac{1}{b e - a f} b \text{EllipticF}[\right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right] \right) \middle/ \right. \\
& \left. \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) - \right. \\
& \left. \left(e h \sqrt{-\frac{(b e - a f) (-f g + e h) \left(-\frac{b}{e+a f} + \frac{1}{e+f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right. \right. \\
& \left. \left. \left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}]\right) \middle/ \left((b e - a f) (-f g + e h) \right) - \frac{1}{b e - a f} b \text{EllipticF}[\right. \right. \\
& \left. \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right] \right) \middle/ \right. \\
& \left. \left. \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) - \right. \right. \\
& \left. \left. \left(1 \middle/ \left(2 d h^2 \sqrt{e + f x} \left(b - \frac{b e}{e + f x} + \frac{a f}{e + f x}\right) \left(d - \frac{d e}{e + f x} + \frac{c f}{e + f x}\right) \sqrt{g + \frac{(e + f x) \left(h - \frac{e h}{e + f x}\right)}{f}} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & (b d f g + b d e h - b c f h - a d f h) \\
 & \sqrt{\left(\left(b - \frac{b e}{e + f x} + \frac{a f}{e + f x}\right) \left(d - \frac{d e}{e + f x} + \frac{c f}{e + f x}\right) \left(h + \frac{f g}{e + f x} - \frac{e h}{e + f x}\right)\right)} \\
 & \sqrt{a + \frac{(e + f x) \left(b - \frac{b e}{e + f x}\right)}{f}} \sqrt{c + \frac{(e + f x) \left(d - \frac{d e}{e + f x}\right)}{f}} \\
 & \left(\left(d f g \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e + f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \left(-\frac{h}{-f g + e h} + \frac{1}{e + f x}\right) \right. \right. \\
 & \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\Bigg] \Bigg) / \\
 & \left(\sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) - \\
 & \left(2 d e h \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e + f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \left(-\frac{h}{-f g + e h} + \frac{1}{e + f x}\right) \right. \\
 & \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\Bigg] \Bigg) / \\
 & \left(\sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) + \\
 & \left(c f h \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e + f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \left(-\frac{h}{-f g + e h} + \frac{1}{e + f x}\right) \right. \\
 & \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\Bigg] \Bigg) / \\
 & \left(\sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) - \left(d e f \right)
 \end{aligned}$$

$$\begin{aligned}
& g \sqrt{-\frac{(b e - a f) (-f g + e h) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \\
& \left(\left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right] \right) / \left(((b e - a f) (-f g + e h)) - \frac{1}{b e - a f} b \text{EllipticF}[\right. \right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right]\right) / \\
& \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) + \left(c f^2 \right. \\
& \left. g \sqrt{-\frac{(b e - a f) (-f g + e h) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right. \\
& \left(\left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right] \right) / \left(((b e - a f) (-f g + e h)) - \frac{1}{b e - a f} b \text{EllipticF}[\right. \right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) (-d g + c h)}{(d e - c f) (-b g + a h)}\right]\right) / \\
& \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) + \left(d e^2 \right. \\
& \left. h \sqrt{-\frac{(b e - a f) (-f g + e h) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right)
\end{aligned}$$

$$\begin{aligned}
 & \left(\left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g + c h)}} \right], \right. \right. \\
 & \left. \left. \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right) / \left((b e - a f) \left(-f g + e h\right) \right) - \frac{1}{b e - a f} b \text{EllipticF}[\\
 & \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g + c h)}} \right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right) / \\
 & \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) - \left(c e f \right. \\
 & \left. h \sqrt{-\frac{(b e - a f) \left(-f g + e h\right) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right. \\
 & \left(\left((-b f g + a f h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g + c h)}} \right], \right. \right. \\
 & \left. \left. \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right) / \left((b e - a f) \left(-f g + e h\right) \right) - \frac{1}{b e - a f} b \text{EllipticF}[\\
 & \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g + c h)}} \right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right) / \\
 & \left(\sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} \right) - \\
 & \left(d (-f g + e h) \left(-\frac{d}{d e - c f} + \frac{h}{-f g + e h}\right) \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e + f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \right. \\
 & \left. \sqrt{-\frac{\left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \left(-\frac{h}{-f g + e h} + \frac{1}{e + f x}\right)}{\left(-\frac{d}{d e - c f} + \frac{h}{-f g + e h}\right)^2}} \text{EllipticPi}\left[-\frac{-d f g + c f h}{(d e - c f) h}, \right. \right. \\
 & \left. \left. \right]
 \end{aligned}$$

$$\begin{aligned} & \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)}\Bigg] \\ & \left.\left(\sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)}\right)\right) \end{aligned}$$

Problem 107: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{3/2}}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 968 leaves, 10 steps):

$$\begin{aligned}
 & \frac{b \sqrt{a+b x} \sqrt{c+d x} \sqrt{g+h x}}{d h \sqrt{e+f x}} - \\
 & \left(b \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a+b x} \sqrt{\frac{(d e - c f) (g + h x)}{(d g - c h) (e + f x)}} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{f g - e h} \sqrt{c + d x}}{\sqrt{d g - c h} \sqrt{e + f x}}\right], \right. \\
 & \left. \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right] \Big/ \left(d f h \sqrt{-\frac{(d e - c f) (a + b x)}{(b c - a d) (e + f x)}} \sqrt{g + h x}\right) + \\
 & \left(b (d e - c f) (b f g + b e h - 2 a f h) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \right. \\
 & \left. \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}]\right) \Big/ \\
 & \left(d f^2 h \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}}\right) + \\
 & \left(b \sqrt{b g - a h} (a d f h - b (d f g + d e h - c f h)) \sqrt{\frac{(f g - e h) (a + b x)}{(b g - a h) (e + f x)}} \sqrt{\frac{(f g - e h) (c + d x)}{(d g - c h) (e + f x)}} \right. \\
 & \left. (e + f x) \operatorname{EllipticPi}\left[\frac{f (b g - a h)}{(b e - a f) h}, \operatorname{ArcSin}\left[\frac{\sqrt{b e - a f} \sqrt{g + h x}}{\sqrt{b g - a h} \sqrt{e + f x}}\right], \frac{(d e - c f) (b g - a h)}{(b e - a f) (d g - c h)}\right]\right) \Big/ \\
 & \left(d f^2 \sqrt{b e - a f} h^2 \sqrt{a + b x} \sqrt{c + d x}\right) - \left(2 \sqrt{b c - a d} \sqrt{-d g + c h} (a + b x) \right. \\
 & \left. \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \operatorname{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \right. \right. \\
 & \left. \left. \operatorname{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right]\right) \Big/ \left(d h \sqrt{c + d x} \sqrt{e + f x}\right)
 \end{aligned}$$

Result (type 4, 6638 leaves):

$$-\frac{1}{d^2} 2 \left(-\left(\left(b (c + d x)^{3/2} \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x} \right)}{d}} \right) \right) \Big/$$

$$\begin{aligned}
& \left(2 f h \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}} \right) + \\
& \left(d (b g - a h) (d g - c h) (b f g + b e h - 2 a f h) \sqrt{c + d x} \right. \\
& \quad \left. \sqrt{\left(\left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x} \right) \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x} \right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x} \right) \right)} \right. \\
& \quad \left. \sqrt{a + \frac{(c+d x) \left(b - \frac{b c}{c+d x}\right)}{d}} \left(d e \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \right. \\
& \quad \left. \left. \left(-\frac{f}{-d e + c f} + \frac{1}{c+d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \right. \\
& \quad \left. \left. \left(-b d g + a d h \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \right. \right. \\
& \quad \left. \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) \right. \Big/ \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\right. \\
& \quad \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) \Big/ \\
& \quad \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x} \right) \left(f + \frac{d e - c f}{c+d x} \right) \left(h + \frac{d g - c h}{c+d x} \right)} \right) - \\
& \quad \left(c f \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \\
& \quad \left. \left(-\frac{f}{-d e + c f} + \frac{1}{c+d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right)
\end{aligned}$$

$$\begin{aligned}
 & \left(\left((-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right) \right. \\
 & \left. / \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \operatorname{EllipticF}[\right. \\
 & \left. \operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right) \right) / \\
 & \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} + \right. \\
 & \left. \left(f \sqrt{\frac{-\frac{b}{b c - a d} + \frac{1}{c+d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \left(-\frac{h}{-d g + c h} + \frac{1}{c + d x}\right) \right. \right. \\
 & \left. \left. \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-d e + c f) \left(-h - \frac{d g}{c+d x} + \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right) \right) / \\
 & \left(\sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) \right) / \\
 & \left(2 f h^2 (f g - e h) \left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \right. \\
 & \left. \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c+d x}\right)}{d}} - \right. \\
 & \left. d (b e - a f) (d e - c f) (b f g + b e h - 2 a f h) \sqrt{c + d x} \right. \\
 & \left. \sqrt{\left(\left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)\right)} \right)
 \end{aligned}$$

$$\begin{aligned}
& \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c+d x}\right)}{d}} \\
& \left(\frac{d g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c-a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}}}{\left(-\frac{f}{-d e + c f} + \frac{1}{c + d x}\right) \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}}} \right. \\
& \left(\left(-b d g + a d h \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \right. \\
& \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \\
& \left(\sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \\
& \left(c h \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c-a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \\
& \left. \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \right. \\
& \left(\left(-b d g + a d h \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \right. \\
& \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\right.
\end{aligned}$$

$$\begin{aligned}
 & \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]\Bigg] \\
 & \left(\sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} + \right. \\
 & \left. h \sqrt{\frac{-\frac{b}{b c-a d} + \frac{1}{c+d x}}{-\frac{b}{b c-a d} + \frac{h}{-d g+c h}}} \sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \left(-\frac{h}{-d g + c h} + \frac{1}{c + d x}\right)\right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-d e + c f) \left(-h - \frac{d g}{c+d x} + \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]\right]\right) \\
 & \left(\sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}\right)\Bigg) \\
 & \left(2 f^2 h (f g - e h) \left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c+d x}\right)}{d}}\right. \\
 & \left. \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c+d x}\right)}{d}}\right) - \\
 & \frac{1}{2 f^2 h^2 \left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c+d x}\right)}{d}}} \\
 & b \frac{(b d f g + b d e h + b c f h - 3 a d f h)}{\sqrt{c + d x}} \\
 & \sqrt{\left(\left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)\right)} \\
 & \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c+d x}\right)}{d}} \\
 & \left(d^2 e g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c-a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}}\right)
 \end{aligned}$$

$$\begin{aligned}
& \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \\
& \left(\left((-b d g + a d h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \right. \right. \right. \\
& \left. \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \\
& \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x} \right) \left(f + \frac{d e - c f}{c + d x} \right) \left(h + \frac{d g - c h}{c + d x} \right)} - \right. \\
& \left. \left(c d f g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \right. \right. \\
& \left. \left. \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \right. \\
& \left(\left((-b d g + a d h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \right. \right. \right. \\
& \left. \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \\
& \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x} \right) \left(f + \frac{d e - c f}{c + d x} \right) \left(h + \frac{d g - c h}{c + d x} \right)} - \right.
\end{aligned}$$

$$\begin{aligned}
 & \left(c d e h \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \right. \\
 & \quad \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \\
 & \quad \left(\left(-b d g + a d h \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \right. \\
 & \quad \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\\
 & \quad \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \\
 & \quad \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x} \right) \left(f + \frac{d e - c f}{c + d x} \right) \left(h + \frac{d g - c h}{c + d x} \right)} + \right. \\
 & \quad \left. \left(c^2 f h \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \right. \right. \\
 & \quad \left. \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \\
 & \quad \left(\left(-b d g + a d h \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \right. \\
 & \quad \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\\
 & \quad \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx} \right) \left(f + \frac{de-cf}{c+dx} \right) \left(h + \frac{dg-ch}{c+dx} \right)} \right) + \\
& \left(dfg \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \left(-\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-de+cf) (-h - \frac{dg}{c+dx} + \frac{ch}{c+dx})}{d (-fg+eh)}} \right], \frac{(bc-ad) (-fg+eh)}{(-de+cf) (-bg+ah)}] \right) / \\
& \left(\sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx} \right) \left(f + \frac{de-cf}{c+dx} \right) \left(h + \frac{dg-ch}{c+dx} \right)} \right) + \\
& \left(deh \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \left(-\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-de+cf) (-h - \frac{dg}{c+dx} + \frac{ch}{c+dx})}{d (-fg+eh)}} \right], \frac{(bc-ad) (-fg+eh)}{(-de+cf) (-bg+ah)}] \right) / \\
& \left(\sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx} \right) \left(f + \frac{de-cf}{c+dx} \right) \left(h + \frac{dg-ch}{c+dx} \right)} \right) - \\
& \left(2cfgh \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \left(-\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-de+cf) (-h - \frac{dg}{c+dx} + \frac{ch}{c+dx})}{d (-fg+eh)}} \right], \frac{(bc-ad) (-fg+eh)}{(-de+cf) (-bg+ah)}] \right) / \\
& \left(\sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx} \right) \left(f + \frac{de-cf}{c+dx} \right) \left(h + \frac{dg-ch}{c+dx} \right)} \right) - \\
& \left(f (-dg+ch) \left(-\frac{f}{-de+cf} + \frac{h}{-dg+ch} \right) \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \right)
\end{aligned}$$

$$\begin{aligned} & \sqrt{-\frac{\left(-\frac{f}{-d e+c f}+\frac{1}{c+d x}\right)\left(-\frac{h}{-d g+c h}+\frac{1}{c+d x}\right)}{\left(-\frac{f}{-d e+c f}+\frac{h}{-d g+c h}\right)^2}} \operatorname{EllipticPi}\left[-\frac{-d f g+d e h}{(-d e+c f) h},\right. \\ & \left.\operatorname{ArcSin}\left[\sqrt{\frac{(-d e+c f)\left(-h-\frac{d g}{c+d x}+\frac{c h}{c+d x}\right)}{d(-f g+e h)}}\right], \frac{(b c-a d)(-f g+e h)}{(-d e+c f)(-b g+a h)}\right] / \\ & \left.\left(\sqrt{\left(b+\frac{-b c+a d}{c+d x}\right)\left(f+\frac{d e-c f}{c+d x}\right)\left(h+\frac{d g-c h}{c+d x}\right)}\right)\right) \end{aligned}$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x}}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 228 leaves, 2 steps) :

$$\begin{aligned} & \left(2 \sqrt{-d g+c h}(a+b x) \sqrt{\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}} \sqrt{\frac{(b g-a h)(e+f x)}{(f g-e h)(a+b x)}}\right. \\ & \left.\operatorname{EllipticPi}\left[-\frac{b(d g-c h)}{(b c-a d) h}, \operatorname{ArcSin}\left[\frac{\sqrt{b c-a d} \sqrt{g+h x}}{\sqrt{-d g+c h} \sqrt{a+b x}}\right], \frac{(b e-a f)(d g-c h)}{(b c-a d)(f g-e h)}\right]\right) / \\ & (\sqrt{b c-a d} h \sqrt{c+d x} \sqrt{e+f x}) \end{aligned}$$

Result (type 4, 584 leaves) :

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}} (c + d x)^{3/2} \right. \right. \\
& \left. \left(a d \sqrt{\frac{(d g - c h) (e + f x)}{(f g - e h) (c + d x)}} (g + h x) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}} \right], \right. \right. \\
& \left. \left. \frac{(b c - a d) (-f g + e h)}{(d e - c f) (b g - a h)} \right] \right) / \left((d g - c h) (c + d x) \sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}} \right) + \\
& \left(b c \sqrt{\frac{(d g - c h) (e + f x)}{(f g - e h) (c + d x)}} (g + h x) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}} \right], \right. \right. \\
& \left. \left. \frac{(b c - a d) (-f g + e h)}{(d e - c f) (b g - a h)} \right] \right) / \left((-d g + c h) (c + d x) \sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}} \right) + \\
& \frac{1}{(d e - c f) h} b (f g - e h) \sqrt{-\frac{(d e - c f) (d g - c h) (e + f x) (g + h x)}{(f g - e h)^2 (c + d x)^2}} \\
& \operatorname{EllipticPi}\left[\frac{d (-f g + e h)}{(d e - c f) h}, \operatorname{ArcSin}\left[\sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}} \right], \right. \right. \\
& \left. \left. \frac{(b c - a d) (-f g + e h)}{(d e - c f) (b g - a h)} \right] \right) / \left(d \sqrt{a + b x} \sqrt{e + f x} \sqrt{g + h x} \right)
\end{aligned}$$

Problem 110: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x)^{3/2} \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 429 leaves, 5 steps):

$$\begin{aligned}
 & - \left(\left(2 b \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \right. \right. \\
 & \quad \left. \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}]\right) / \right. \\
 & \quad \left. \left((b c - a d) (b e - a f) \sqrt{b g - a h} \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \right) \right) - \\
 & \quad \left(2 d \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], \right. \\
 & \quad \left. \left. -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}\right] \right) / \\
 & \quad \left. \left((b c - a d) \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \right) \right)
 \end{aligned}$$

Result (type 4, 3247 leaves):

$$\begin{aligned}
 & - \frac{2 b^2 \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a + b x}} - \frac{1}{d (b c - a d) (b e - a f) (b g - a h)} \\
 & 2 \left(- \left(\left(b (c + d x)^{3/2} \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x} \right)}{d}} \right) \right. \right. \\
 & \quad \left. \left. \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c + d x} \right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c + d x} \right)}{d}} \right) + \right. \\
 & \quad \left. \left(1 / \left((f g - e h) \left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c + d x} \right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c + d x} \right)}{d}} \right) \right. \right. \\
 & \quad \left. \left. (b c - a d) f (b g - a h) (-d g + c h) \sqrt{c + d x} \sqrt{\left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right)} \right. \right. \\
 & \quad \left. \left. \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x} \right)}{d}} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \left(\left(d e \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x}\right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \right. \\
& \left. \left. \left((-b d g + a d h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-d g + c h)}{(-d e + c f) (-b g + a h)}\right] \right) \middle/ \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right] \right) \middle/ \right. \\
& \left. \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} - c f \right. \right. \\
& \left. \left. \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x}\right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \right. \\
& \left. \left. \left((-b d g + a d h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right] \right) \middle/ \left((b c - a d) (-d g + c h) \right) - \frac{1}{b c - a d} b \text{EllipticF}[\right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right] \right) \middle/ \right. \\
& \left. \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} + \right. \right. \\
& \left. \left. f \sqrt{\frac{-\frac{b}{b c - a d} + \frac{1}{c + d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{-\frac{\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \left(-\frac{h}{-d g + c h} + \frac{1}{c + d x}\right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-d e + c f) \left(-h - \frac{d g}{c+d x} + \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) \left(-f g + e h\right)}{(-d e + c f) \left(-b g + a h\right)}]\Bigg] \\
 & \left(\sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}\right) - \\
 & \left(1 / \left(\left(f g - e h\right) \left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}}\right)\right) - \\
 & (b c - a d) (b e - a f) (-d e + c f) h \sqrt{c + d x} \\
 & \sqrt{\left(\left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)\right)} \\
 & \sqrt{a + \frac{(c+d x) \left(b - \frac{b c}{c+d x}\right)}{d}} \\
 & \left(d g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c+d x}\right) \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}}\right. \\
 & \left.\left((-b d g + a d h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) \left(-f g + e h\right)}{(-d e + c f) \left(-b g + a h\right)}]\right)\right) / \left((b c - a d) (-d g + c h)\right) - \frac{1}{b c - a d} b \text{EllipticF}[\\
 & \text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) \left(-f g + e h\right)}{(-d e + c f) \left(-b g + a h\right)}]\Bigg) \\
 & \left(\sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}\right) - \left(c h\right. \\
 & \left.\sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c+d x}\right) \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}}\right)
 \end{aligned}$$

$$\begin{aligned}
& \left(\left((-b d g + a d h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) (h + \frac{d g}{c+d x} - \frac{c h}{c+d x})}{d (-f g + e h)}} \right], \right. \right. \\
& \left. \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \left(((b c - a d) (-d g + c h)) - \frac{1}{b c - a d} b \text{EllipticF}[\right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{(d e - c f) (h + \frac{d g}{c+d x} - \frac{c h}{c+d x})}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \\
& \left(\sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x} \right) \left(f + \frac{d e - c f}{c + d x} \right) \left(h + \frac{d g - c h}{c + d x} \right)} + \right. \\
& \left. \left(h \sqrt{\frac{-\frac{b}{b c - a d} + \frac{1}{c+d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \left(-\frac{h}{-d g + c h} + \frac{1}{c + d x} \right) \right. \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-d e + c f) (-h - \frac{d g}{c+d x} + \frac{c h}{c+d x})}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) / \right. \\
& \left. \left(\sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x} \right) \left(f + \frac{d e - c f}{c + d x} \right) \left(h + \frac{d g - c h}{c + d x} \right)} \right) \right)
\end{aligned}$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{3/2} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 786 leaves, ? steps):

$$\begin{aligned}
& - \frac{2 d^3 \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (d e-c f) (d g-c h) \sqrt{c+d x}} - \frac{2 b^3 \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (b e-a f) (b g-a h) \sqrt{a+b x}} + \\
& \left(2 b (a^2 d^2 f h - a b d^2 (f g + e h) + b^2 (2 d^2 e g + c^2 f h - c d (f g + e h))) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \right) / \\
& \left((b c-a d)^2 (b e-a f) (d e-c f) (b g-a h) (d g-c h) \sqrt{a+b x} \right) - \\
& \left(2 \sqrt{f g - e h} (a^2 d^2 f h - a b d^2 (f g + e h) + b^2 (2 d^2 e g + c^2 f h - c d (f g + e h))) \sqrt{c+d x} \right. \\
& \left. \sqrt{- \frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}} \right], - \frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}] \right) / \\
& \left((b c-a d)^2 (b e-a f) (d e-c f) \sqrt{b g-a h} (d g-c h) \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)} \sqrt{g+h x}} \right) - \\
& \left(4 b d \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}} \sqrt{g+h x} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}} \right], - \frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}] \right) / \\
& \left((b c-a d)^2 \sqrt{b g-a h} \sqrt{f g-e h} \sqrt{c+d x} \sqrt{- \frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}} \right)
\end{aligned}$$

Result (type 4, 7061 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \\
& \left(\frac{1}{\left(c - \frac{a d}{b}\right) (a+b x)} \left(- \frac{2 b^3 c d^2 e g}{(b c-a d)^2 (b e-a f) (-d e+c f) (b g-a h) (-d g+c h)} \right. \right. - \\
& \left. \left. \frac{2 a b^2 d^3 e g}{(b c-a d)^2 (b e-a f) (-d e+c f) (b g-a h) (-d g+c h)} + \right. \right. \\
& \left. \left. \frac{2 b^3 c^2 d f g}{(b c-a d)^2 (b e-a f) (-d e+c f) (b g-a h) (-d g+c h)} + \right. \right. \\
& \left. \left. \frac{2 a^2 b d^3 f g}{(b c-a d)^2 (b e-a f) (-d e+c f) (b g-a h) (-d g+c h)} + \right. \right. \\
& \left. \left. \frac{2 b^3 c^2 d e h}{(b c-a d)^2 (b e-a f) (-d e+c f) (b g-a h) (-d g+c h)} + \right. \right. \\
& \left. \left. \frac{2 a^2 b d^3 e h}{(b c-a d)^2 (b e-a f) (-d e+c f) (b g-a h) (-d g+c h)} - \right. \right. \\
& \left. \left. \frac{2 b^3 c^3 f h}{(b c-a d)^2 (b e-a f) (-d e+c f) (b g-a h) (-d g+c h)} \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{2 a^3 d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{1}{b} \\
& a \left(- \frac{4 b^3 d^3 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \right. \\
& \frac{2 b^3 c d^2 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \frac{2 a b^2 d^3 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \frac{2 b^3 c d^2 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \frac{2 a b^2 d^3 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \frac{2 b^3 c^2 d f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \left. \frac{2 a^2 b d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} \right) + \\
& \frac{1}{\left(a - \frac{b c}{d}\right) (c + d x)} \left(- \frac{2 b^3 c d^2 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \right. \\
& \frac{2 a b^2 d^3 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \frac{2 b^3 c^2 d f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \frac{2 a^2 b d^3 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \frac{2 b^3 c^2 d e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \frac{2 a^2 b d^3 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \frac{2 b^3 c^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \frac{2 a^3 d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{1}{d} \\
& c \left(- \frac{4 b^3 d^3 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \right. \\
& \frac{2 b^3 c d^2 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} +
\end{aligned}$$

$$\begin{aligned}
 & \frac{2 a b^2 d^3 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
 & \frac{2 b^3 c d^2 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
 & \frac{2 a b^2 d^3 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
 & \frac{2 b^3 c^2 d f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
 & \left. \left. \left. \left. \frac{2 a^2 b d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} \right) \right) \right) - \\
 & \frac{1}{b^2 (-b c + a d)^2 (-b e + a f) (-d e + c f) (-b g + a h) (-d g + c h)} \\
 & \frac{2}{\left(\left((-2 b^2 d^2 e g + b^2 c d f g + a b d^2 f g + b^2 c d e h + a b d^2 e h - b^2 c^2 f h - a^2 d^2 f h) \right. \right.} \\
 & \left. \left. \left((a + b x)^{5/2} \left(d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left(f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \right) \right) / \right. \\
 & \left. \left. \left. \left(\sqrt{c + \frac{(a + b x) \left(d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left(f - \frac{a f}{a + b x} \right)}{b}} \sqrt{g + \frac{(a + b x) \left(h - \frac{a h}{a + b x} \right)}{b}} \right) + \right. \right. \\
 & \left. \left. \left. \left. \frac{1}{\sqrt{c + \frac{(a + b x) \left(d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left(f - \frac{a f}{a + b x} \right)}{b}} \sqrt{g + \frac{(a + b x) \left(h - \frac{a h}{a + b x} \right)}{b}}} \right. \right. \right. \\
 & \left. \left. \left. \left. (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sqrt{\left(\left(d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left(f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \right)} \right. \right. \right. \\
 & \left. \left. \left. \left. \left(2 b^2 d^2 e g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \left(-\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left(\left(b d g - b c h \right) \text{EllipticE}[\right. \right. \right. \right. \\
 \end{aligned}$$

$$\begin{aligned}
& \left. \text{ArcSin} \left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right\} \\
& \left. \left((b c - a d) (b g - a h) \right) \right\} - \frac{1}{-b c + a d} d \text{EllipticF} [\text{ArcSin} [\\
& \left. \left. \sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right] \right\} \\
& \left. \left(\sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) - \right. \\
& \left. \left. \left. b^2 c d f g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \left(-\frac{f}{-b e + a f} + \frac{1}{a + b x}\right) \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left(-\left(b d g - b c h\right) \text{EllipticE} [\right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \text{ArcSin} \left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right] \right\} \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left((b c - a d) (b g - a h) \right) \right\} - \frac{1}{-b c + a d} d \text{EllipticF} [\text{ArcSin} [\right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right] \right\} \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left(\sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) - \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left. a b d^2 f g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \left(-\frac{f}{-b e + a f} + \frac{1}{a + b x}\right) \right. \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{-\frac{h}{-b g+a h} + \frac{1}{a+b x}}{\frac{f}{-b e+a f} - \frac{h}{-b g+a h}}} \left(- \left(\left((b d g - b c h) \text{EllipticE} \left[\right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \left. \left. \left(\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)} \right], \frac{(-b c + a d) \left(-f g + e h \right)}{(-b e + a f) \left(-d g + c h \right)} \right] \right) \right. \right. \\
 & \left. \left. \left. \left. \left. \left. \left((b c - a d) \left(b g - a h \right) \right) \right) - \frac{1}{-b c + a d} d \text{EllipticF} \left[\text{ArcSin} \left[\right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \left. \left. \left(\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)} \right], \frac{(-b c + a d) \left(-f g + e h \right)}{(-b e + a f) \left(-d g + c h \right)} \right] \right) \right] \right) \right. \right. \\
 & \left(\sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x} \right) \left(f + \frac{b e - a f}{a + b x} \right) \left(h + \frac{b g - a h}{a + b x} \right)} \right) - \\
 & \left(b^2 c d e h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \left(-\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \right. \\
 & \left. \sqrt{\frac{-\frac{h}{-b g+a h} + \frac{1}{a+b x}}{\frac{f}{-b e+a f} - \frac{h}{-b g+a h}}} \left((b d g - b c h) \text{EllipticE} \left[\right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \left. \left. \left(\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)} \right], \frac{(-b c + a d) \left(-f g + e h \right)}{(-b e + a f) \left(-d g + c h \right)} \right] \right) \right. \right. \\
 & \left. \left. \left. \left. \left. \left. \left((b c - a d) \left(b g - a h \right) \right) \right) - \frac{1}{-b c + a d} d \text{EllipticF} \left[\text{ArcSin} \left[\right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \left. \left. \left(\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)} \right], \frac{(-b c + a d) \left(-f g + e h \right)}{(-b e + a f) \left(-d g + c h \right)} \right] \right) \right] \right) \right] \right) \right. \right. \\
 & \left(\sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x} \right) \left(f + \frac{b e - a f}{a + b x} \right) \left(h + \frac{b g - a h}{a + b x} \right)} \right) -
 \end{aligned}$$

$$\begin{aligned}
& \left(a b d^2 e h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \left(-\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \right. \\
& \left. \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left(- \left(\begin{array}{l} (b d g - b c h) \text{EllipticE} \left[\right. \\ \left. \sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) \right. \\
& \left. \left((b c - a d) (b g - a h) \right) \right) - \frac{1}{-b c + a d} d \text{EllipticF} [\text{ArcSin} \left[\right. \\
& \left. \left. \sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]] \right) / \\
& \left(\sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x} \right) \left(f + \frac{b e - a f}{a+b x} \right) \left(h + \frac{b g - a h}{a+b x} \right)} + \right. \\
& \left. \left(b^2 c^2 f h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \left(-\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \right. \right. \\
& \left. \left. \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left(- \left(\begin{array}{l} (b d g - b c h) \text{EllipticE} \left[\right. \\ \left. \sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) \right) \right. \\
& \left. \left((b c - a d) (b g - a h) \right) \right) - \frac{1}{-b c + a d} d \text{EllipticF} [\text{ArcSin} \left[\right. \\
& \left. \left. \sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
 & \left(\sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) + \\
 & \left(a^2 d^2 f h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c+a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \left(-\frac{f}{-b e+a f} + \frac{1}{a+b x}\right) \right. \\
 & \left. \sqrt{\frac{-\frac{h}{-b g+a h} + \frac{1}{a+b x}}{\frac{f}{-b e+a f} - \frac{h}{-b g+a h}}} \left(\left(b d g - b c h \right) \text{EllipticE} \left[\right. \right. \right. \\
 & \left. \left. \left. \frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) / \\
 & \left. \left((b c - a d) (b g - a h) \right) \right) - \frac{1}{-b c + a d} d \text{EllipticF} \left[\text{ArcSin} \left[\right. \right. \\
 & \left. \left. \frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) / \\
 & \left(\sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) + \\
 & \left(b d^2 f g \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right) \right. \\
 & \left. \text{EllipticF} \left[\text{ArcSin} \left[\right. \right. \right. \\
 & \left. \left. \left. \frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) / \\
 & \left(\sqrt{\frac{-\frac{h}{-b g+a h} + \frac{1}{a+b x}}{\frac{f}{-b e+a f} - \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) + \\
 & \left(b d^2 e h \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}]\Bigg] \\
& \left(\sqrt{\frac{-\frac{h}{-b g+a h} + \frac{1}{a+b x}}{\frac{f}{-b e+a f} - \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) - \\
& \left(b c d f h \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right)\right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}]\Bigg] \\
& \left(\sqrt{\frac{-\frac{h}{-b g+a h} + \frac{1}{a+b x}}{\frac{f}{-b e+a f} - \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) - \\
& \left(a d^2 f h \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{\frac{-\frac{f}{-b e+a f} + \frac{1}{a+b x}}{-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}}} \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right)\right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}]\Bigg] \\
& \left. \left(\sqrt{\frac{-\frac{h}{-b g+a h} + \frac{1}{a+b x}}{\frac{f}{-b e+a f} - \frac{h}{-b g+a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) \right)
\end{aligned}$$

Problem 112: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 (e + f x)^n}{(a + b x) (c + d x)} dx$$

Optimal (type 5, 319 leaves, 8 steps):

$$\begin{aligned} & \frac{e^2 (e + f x)^{1+n}}{b d f^3 (1+n)} + \frac{(b c + a d) e (e + f x)^{1+n}}{b^2 d^2 f^2 (1+n)} + \frac{(b^2 c^2 + a b c d + a^2 d^2) (e + f x)^{1+n}}{b^3 d^3 f (1+n)} - \frac{2 e (e + f x)^{2+n}}{b d f^3 (2+n)} - \\ & \frac{(b c + a d) (e + f x)^{2+n}}{b^2 d^2 f^2 (2+n)} + \frac{(e + f x)^{3+n}}{b d f^3 (3+n)} - \frac{a^4 (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b (e+f x)}{b e-a f}]}{b^3 (b c - a d) (b e - a f) (1+n)} + \\ & \frac{c^4 (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{d (e+f x)}{d e-c f}]}{d^3 (b c - a d) (d e - c f) (1+n)} \end{aligned}$$

Result (type 6, 262 leaves):

$$\begin{aligned} & \frac{6}{5} e x^5 (e + f x)^n \left(\left(a b \text{AppellF1}[5, -n, 1, 6, -\frac{f x}{e}, -\frac{b x}{a}] \right) \right. \\ & \left. \left((b c - a d) (a + b x) \left(6 a e \text{AppellF1}[5, -n, 1, 6, -\frac{f x}{e}, -\frac{b x}{a}] + a f n x \right. \right. \right. \\ & \left. \left. \left. \text{AppellF1}[6, 1-n, 1, 7, -\frac{f x}{e}, -\frac{b x}{a}] - b e x \text{AppellF1}[6, -n, 2, 7, -\frac{f x}{e}, -\frac{b x}{a}] \right) \right) + \\ & \left(c d \text{AppellF1}[5, -n, 1, 6, -\frac{f x}{e}, -\frac{d x}{c}] \right) \left/ \left((-b c + a d) (c + d x) \right. \right. \\ & \left. \left(6 c e \text{AppellF1}[5, -n, 1, 6, -\frac{f x}{e}, -\frac{d x}{c}] + c f n x \text{AppellF1}[6, 1-n, 1, 7, -\frac{f x}{e}, -\frac{d x}{c}] - \right. \right. \\ & \left. \left. d e x \text{AppellF1}[6, -n, 2, 7, -\frac{f x}{e}, -\frac{d x}{c}] \right) \right) \right) \end{aligned}$$

Problem 113: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 (e + f x)^n}{(a + b x) (c + d x)} dx$$

Optimal (type 5, 216 leaves, 6 steps):

$$\begin{aligned} & -\frac{e (e + f x)^{1+n}}{b d f^2 (1+n)} - \frac{(b c + a d) (e + f x)^{1+n}}{b^2 d^2 f (1+n)} + \frac{(e + f x)^{2+n}}{b d f^2 (2+n)} + \\ & \frac{a^3 (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b (e+f x)}{b e-a f}]}{b^2 (b c - a d) (b e - a f) (1+n)} - \\ & \frac{c^3 (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{d (e+f x)}{d e-c f}]}{d^2 (b c - a d) (d e - c f) (1+n)} \end{aligned}$$

Result (type 6, 262 leaves):

$$\begin{aligned} & \frac{5}{4} e x^4 (e + f x)^n \left(\left(a b \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{b x}{a}] \right) \right. \\ & \quad \left. \left((b c - a d) (a + b x) \left(5 a e \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{b x}{a}] + a f n x \right. \right. \right. \\ & \quad \left. \left. \left. \text{AppellF1}[5, 1 - n, 1, 6, -\frac{f x}{e}, -\frac{b x}{a}] - b e x \text{AppellF1}[5, -n, 2, 6, -\frac{f x}{e}, -\frac{b x}{a}] \right) \right) + \\ & \quad \left(c d \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{d x}{c}] \right) \left/ \left((-b c + a d) (c + d x) \right. \right. \\ & \quad \left. \left. \left(5 c e \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{d x}{c}] + c f n x \text{AppellF1}[5, 1 - n, 1, 6, -\frac{f x}{e}, -\frac{d x}{c}] - \right. \right. \right. \\ & \quad \left. \left. \left. d e x \text{AppellF1}[5, -n, 2, 6, -\frac{f x}{e}, -\frac{d x}{c}] \right) \right) \right) \end{aligned}$$

Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x) (e + f x)}{g + h x} dx$$

Optimal (type 5, 134 leaves, 2 steps):

$$\begin{aligned} & - \left(\left((a + b x)^{1+m} (a d f h + b (d f g - d e h - c f h) (2 + m) - b d f h (1 + m) x) \right) \right/ \left(b^2 h^2 (1 + m) (2 + m) \right) + \\ & \left((d g - c h) (f g - e h) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1 + m, 2 + m, -\frac{h (a + b x)}{b g - a h}] \right) \right/ \\ & \left(h^2 (b g - a h) (1 + m) \right) \end{aligned}$$

Result (type 6, 317 leaves):

$$\begin{aligned} & \frac{1}{6} (a + b x)^m \left(\left(9 a (d e + c f) g x^2 \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] \right) \right/ \\ & \left((g + h x) \left(3 a g \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] + b g m x \right. \right. \\ & \left. \left. \text{AppellF1}[3, 1 - m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] - a h x \text{AppellF1}[3, -m, 2, 4, -\frac{b x}{a}, -\frac{h x}{g}] \right) \right) + \\ & \left(8 a d f g x^3 \text{AppellF1}[3, -m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] \right) \left/ \left((g + h x) \right. \right. \\ & \left. \left. \left(4 a g \text{AppellF1}[3, -m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] + b g m x \text{AppellF1}[4, 1 - m, 1, 5, -\frac{b x}{a}, -\frac{h x}{g}] \right) - \right. \right. \\ & \left. \left. a h x \text{AppellF1}[4, -m, 2, 5, -\frac{b x}{a}, -\frac{h x}{g}] \right) \right) + \\ & \frac{6 c e \left(\frac{h (a + b x)}{b (g + h x)} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b g - a h}{b g + b h x}]}{h m} \end{aligned}$$

Problem 121: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)}{(e+f x) (g+h x)} dx$$

Optimal (type 5, 140 leaves, 3 steps):

$$-\frac{\frac{(d e - c f) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1 + m, 2 + m, -\frac{f (a + b x)}{b e - a f}]}{(b e - a f) (f g - e h) (1 + m)} + \frac{(d g - c h) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1 + m, 2 + m, -\frac{h (a + b x)}{b g - a h}]}{(b g - a h) (f g - e h) (1 + m)}}{}$$

Result (type 6, 390 leaves):

$$\begin{aligned} & \frac{1}{2} (a + b x)^m \\ & \left(3 a d x^2 \left(\left(e f \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{f x}{e}] \right) / \left((f g - e h) (e + f x) \left(3 a e \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{f x}{e}] + b e m x \text{AppellF1}[3, 1 - m, 1, 4, -\frac{b x}{a}, -\frac{f x}{e}] - a f x \text{AppellF1}[3, -m, 2, 4, -\frac{b x}{a}, -\frac{f x}{e}] \right) \right) + \left(g h \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] \right) / \left((-f g + e h) (g + h x) \left(3 a g \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] + b g m x \text{AppellF1}[3, 1 - m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] - a h x \text{AppellF1}[3, -m, 2, 4, -\frac{b x}{a}, -\frac{h x}{g}] \right) \right) \right) + \\ & \frac{1}{f g m - e h m} \left(2 c \left(\frac{f (a + b x)}{b (e + f x)} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b e - a f}{b e + b f x}] - 2 c \left(\frac{h (a + b x)}{b (g + h x)} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b g - a h}{b g + b h x}] \right) \end{aligned}$$

Problem 123: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m (e + f x)^n}{(a + b x) (c + d x)} dx$$

Optimal (type 6, 140 leaves, 6 steps):

$$\begin{aligned} & \frac{b x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e} \right)^{-n} \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{b x}{a}]}{a (b c - a d) (1 + m)} - \\ & \frac{d x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e} \right)^{-n} \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{d x}{c}]}{c (b c - a d) (1 + m)} \end{aligned}$$

Result (type 6, 309 leaves):

$$\begin{aligned} & \frac{1}{1+m} e^{(2+m)} x^{1+m} (e+fx)^n \\ & \left(- \left(\left(ab \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{fx}{e}, -\frac{bx}{a}] \right) \middle/ \left((-bc+ad)(a+bx) \right. \right. \right. \\ & \left. \left. \left. \left(ae^{(2+m)} \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{fx}{e}, -\frac{bx}{a}] + x \left(af^n \text{AppellF1}[2+m, 1-n, 1, 3+m, -\frac{fx}{e}, -\frac{bx}{a}] - b e \text{AppellF1}[2+m, -n, 2, 3+m, -\frac{fx}{e}, -\frac{bx}{a}] \right) \right) \right) \right. \\ & \left. \left(cd \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{fx}{e}, -\frac{dx}{c}] \right) \middle/ \left((bc-ad)(c+dx) \right. \right. \\ & \left. \left. \left(ce^{(2+m)} \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{fx}{e}, -\frac{dx}{c}] + x \left(cf^n \text{AppellF1}[2+m, 1-n, 1, 3+m, -\frac{fx}{e}, -\frac{dx}{c}] - de \text{AppellF1}[2+m, -n, 2, 3+m, -\frac{fx}{e}, -\frac{dx}{c}] \right) \right) \right) \right) \end{aligned}$$

Problem 124: Result unnecessarily involves higher level functions.

$$\int (a+bx)^m (c+dx)^n (e+fx) (g+hx) dx$$

Optimal (type 5, 266 leaves, 3 steps):

$$\begin{aligned} & - \left(\left((a+bx)^{1+m} (c+dx)^{1+n} (bcfh(2+m) + adfh(2+n) - \right. \right. \\ & \left. \left. bd(fg+eh)(3+m+n) - bdfh(2+m+n)x) \right) \middle/ (b^2 d^2 (2+m+n) (3+m+n)) \right) + \\ & \left((a^2 d^2 fh(1+n) (2+n) + abd(1+n) (2cfh(1+m) - d(fg+eh)(3+m+n)) + \right. \\ & \left. b^2 (c^2 fh(1+m) (2+m) - cd(fg+eh)(1+m) (3+m+n) + d^2 eg(2+m+n) (3+m+n))) \right. \\ & \left. \left((a+bx)^{1+m} (c+dx)^n \left(\frac{b(c+dx)}{bc-ad} \right)^{-n} \text{Hypergeometric2F1}[1+m, -n, 2+m, -\frac{d(a+bx)}{bc-ad}] \right) \middle/ \right. \\ & \left. (b^3 d^2 (1+m) (2+m+n) (3+m+n)) \right) \end{aligned}$$

Result (type 6, 335 leaves):

$$\begin{aligned} & \frac{1}{3} (a+bx)^m (c+dx)^n \left(\left(9ac(fg+eh)x^2 \text{AppellF1}[2, -m, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}] \right) \middle/ \right. \\ & \left(2 \left(3ac \text{AppellF1}[2, -m, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}] + bc m x \text{AppellF1}[3, 1-m, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}] \right) + \right. \\ & \left. a d n x \text{AppellF1}[3, -m, 1-n, 4, -\frac{bx}{a}, -\frac{dx}{c}] \right) + \\ & \left(4acfhx^3 \text{AppellF1}[3, -m, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}] \right) \middle/ \\ & \left(4ac \text{AppellF1}[3, -m, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}] + bc m x \text{AppellF1}[4, 1-m, -n, 5, -\frac{bx}{a}, -\frac{dx}{c}] \right. + \\ & \left. a d n x \text{AppellF1}[4, -m, 1-n, 5, -\frac{bx}{a}, -\frac{dx}{c}] \right) + \frac{1}{d(1+n)} \\ & 3eg \left(\frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \text{Hypergeometric2F1}[-m, 1+n, 2+n, \frac{b(c+dx)}{bc-ad}] \end{aligned}$$

Problem 125: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 245 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{12 b^2 d^2} (a + b x)^{1+m} (c + d x)^{2-m} (4 b d (f g + e h) - a d f h (3 - m) - b c f h (2 + m) + 3 b d f h x) + \\ & \frac{1}{12 b^4 d^2 (1 + m)} (b c - a d) (a^2 d^2 f h (6 - 5 m + m^2) - 2 a b d (2 - m) (2 d (f g + e h) - c f h (1 + m)) + \\ & b^2 (12 d^2 e g - 4 c d (f g + e h) (1 + m) + c^2 f h (2 + 3 m + m^2))) (a + b x)^{1+m} \\ & (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[-1 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right] \end{aligned}$$

Result (type 6, 1043 leaves):

$$\begin{aligned} & \left(3 a c d e g x^2 (a + b x)^m (c + d x)^{-m} \text{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \\ & \left(6 a c \text{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + \right. \\ & \left. 2 m x \left(b c \text{AppellF1}\left[3, 1 - m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] - a d \text{AppellF1}\left[3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) \right) + \\ & \left(3 a c^2 f g x^2 (a + b x)^m (c + d x)^{-m} \text{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \\ & \left(6 a c \text{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + \right. \\ & \left. 2 m x \left(b c \text{AppellF1}\left[3, 1 - m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] - a d \text{AppellF1}\left[3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) \right) + \\ & \left(3 a c^2 e h x^2 (a + b x)^m (c + d x)^{-m} \text{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \\ & \left(6 a c \text{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + \right. \\ & \left. 2 m x \left(b c \text{AppellF1}\left[3, 1 - m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] - a d \text{AppellF1}\left[3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) \right) + \\ & \left(4 a c d f g x^3 (a + b x)^m (c + d x)^{-m} \text{AppellF1}\left[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \\ & \left(12 a c \text{AppellF1}\left[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + 3 b c m x \text{AppellF1}\left[4, 1 - m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] - \right. \\ & \left. 3 a d m x \text{AppellF1}\left[4, -m, 1 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) + \\ & \left(4 a c d e h x^3 (a + b x)^m (c + d x)^{-m} \text{AppellF1}\left[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \\ & \left(12 a c \text{AppellF1}\left[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + 3 b c m x \text{AppellF1}\left[4, 1 - m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] - \right. \\ & \left. 3 a d m x \text{AppellF1}\left[4, -m, 1 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) + \end{aligned}$$

$$\begin{aligned}
& \left(4 a c^2 f h x^3 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left(12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \\
& \left. 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \\
& \left(5 a c d f h x^4 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left(20 a c \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + 4 b c m x \text{AppellF1}[5, 1-m, m, 6, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \\
& \left. 4 a d m x \text{AppellF1}[5, -m, 1+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\
& \frac{1}{d (-1+m)} c e g (c + d x)^{1-m} \left(a - \frac{b c}{d} + \frac{b (c + d x)}{d} \right)^m \left(1 + \frac{b (c + d x)}{\left(a - \frac{b c}{d} \right) d} \right)^{-m} \\
& \text{Hypergeometric2F1}[1-m, -m, 2-m, -\frac{b (c + d x)}{\left(a - \frac{b c}{d} \right) d}]
\end{aligned}$$

Problem 126: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^{-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 235 leaves, 3 steps) :

$$\begin{aligned}
& \frac{1}{6 b^2 d^2} (a + b x)^{1+m} (c + d x)^{1-m} (3 b d (f g + e h) - a d f h (2 - m) - b c f h (2 + m) + 2 b d f h x) + \\
& \frac{1}{6 b^3 d^2 (1 + m)} (a^2 d^2 f h (2 - 3 m + m^2) - a b d (1 - m) (3 d (f g + e h) - 2 c f h (1 + m))) + \\
& b^2 (6 d^2 e g - 3 c d (f g + e h) (1 + m) + c^2 f h (2 + 3 m + m^2)) (a + b x)^{1+m} \\
& (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 324 leaves) :

$$\begin{aligned}
 & (a + b x)^m (c + d x)^{-m} \left(\left(3 a c (f g + e h) x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
 & \left. \left(6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \right. \right. \\
 & \left. \left(b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
 & \left(4 a c f h x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
 & \left. \left(12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \right. \\
 & \left. \left. 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \frac{1}{d (-1+m)} \right. \\
 & \left. e g \left(\frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b (c + d x)}{b c - a d}] \right)
 \end{aligned}$$

Problem 127: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^{-1-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 261 leaves, 3 steps) :

$$\begin{aligned}
 & \left((a + b x)^{1+m} (c + d x)^{-m} \right. \\
 & \left. (2 b d^2 e g + b c^2 f h (2+m) - c d (2 b (f g + e h) + a f h m) + d (b c - a d) f h m x) \right) / \\
 & (2 b d^2 (b c - a d) m) - \left((b^2 c^2 f h (1+m) (2+m) - 2 b c d (1+m) (b f g + b e h + a f h m) + \right. \\
 & \left. d^2 (2 b^2 e g + 2 a b (f g + e h) m - a^2 f h (1-m) m) \right) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \\
 & \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}] \Big) / (2 b^2 d^2 (b c - a d) m (1+m))
 \end{aligned}$$

Result (type 6, 346 leaves) :

$$\frac{1}{6} (a + b x)^m (c + d x)^{-m} \left(\left(9 a c (f g + e h) x^2 \text{AppellF1}[2, -m, 1 + m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right.$$

$$\left((c + d x) \left(3 a c \text{AppellF1}[2, -m, 1 + m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[3, 1 - m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \right.$$

$$\left(8 a c f h x^3 \text{AppellF1}[3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) /$$

$$\left((c + d x) \left(4 a c \text{AppellF1}[3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[4, 1 - m, 1 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \right.$$

$$\left. \left. 6 e g \left(\frac{d (a + b x)}{-b c + a d} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b (c + d x)}{b c - a d}] \right) / d m \right)$$

Problem 128: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^{-2-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 203 leaves, 3 steps):

$$\left((a + b x)^{1+m} (c + d x)^{-1-m} \right.$$

$$\left(b d^2 e g + b c^2 f h (2 + m) - c d (b (f g + e h) + a f h (1 + m)) + d (b c - a d) f h (1 + m) x \right) /$$

$$(b d^2 (b c - a d) (1 + m)) - \frac{1}{b d^3 m} (a d f h m + b (d (f g + e h) - c f h (2 + m))) (a + b x)^m$$

$$\left. \left(-\frac{d (a + b x)}{b c - a d} \right)^{-m} (c + d x)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b (c + d x)}{b c - a d}] \right)$$

Result (type 6, 303 leaves):

$$\frac{1}{6} (a + b x)^m (c + d x)^{-2-m}$$

$$\left(\frac{6 e g (a + b x) (c + d x)}{(b c - a d) (1 + m)} - \left(9 a c (f g + e h) x^2 \text{AppellF1}[2, -m, 2 + m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right.$$

$$\left(-3 a c \text{AppellF1}[2, -m, 2 + m, 3, -\frac{b x}{a}, -\frac{d x}{c}] - b c m x \text{AppellF1}[3, 1 - m, 2 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + a d (2 + m) x \text{AppellF1}[3, -m, 3 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) -$$

$$\left(8 a c f h x^3 \text{AppellF1}[3, -m, 2 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) /$$

$$\left. \left(-4 a c \text{AppellF1}[3, -m, 2 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - b c m x \text{AppellF1}[4, 1 - m, 2 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d (2 + m) x \text{AppellF1}[4, -m, 3 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right)$$

Problem 129: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-3-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 246 leaves, 3 steps) :

$$\begin{aligned} & - \left(\left((a + b x)^{1+m} (c + d x)^{-2-m} (a^2 b c f h m - a^3 d f h (1+m) - b^3 c e g (2+m) + \right. \right. \\ & \quad a b^2 (c (f g + e h) + d e g (1+m)) - b (a^2 d f h (3+2m) + b^2 (d e g + c (f g + e h) (1+m)) - \\ & \quad a b (2 c f h (1+m) + d (f g + e h) (2+m))) x) \Big) / \left(b^2 (b c - a d)^2 (1+m) (2+m) \right) + \\ & \left(f h (a + b x)^{3+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[3+m, 3+m, 4+m, -\frac{d (a + b x)}{b c - a d}] \right) / \\ & \left. \left. \left((b c - a d)^3 (3+m) \right) \right) \right)$$

Result (type 6, 633 leaves) :

$$\begin{aligned} & \frac{1}{3} (a + b x)^m (c + d x)^{-3-m} \left(\left(3 f g \left(\frac{c (a + b x)}{a (c + d x)} \right)^{-m} (c + d x) \right. \right. \\ & \quad \left(b^2 c^2 (1+m) x^2 \left(\frac{c (a + b x)}{a (c + d x)} \right)^m - a b c x \left(\frac{c (a + b x)}{a (c + d x)} \right)^m (-c m + d (2+m) x) + \right. \\ & \quad \left. a^2 \left(d^2 x^2 - c^2 \left(-1 + \left(\frac{c (a + b x)}{a (c + d x)} \right)^m \right) - c d x \left(-2 + 2 \left(\frac{c (a + b x)}{a (c + d x)} \right)^m + m \left(\frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) \right) \Big) \Big) / \\ & \left(c (b c - a d)^2 (1+m) (2+m) \right) + \left(3 e h \left(\frac{c (a + b x)}{a (c + d x)} \right)^{-m} (c + d x) \right. \\ & \quad \left(b^2 c^2 (1+m) x^2 \left(\frac{c (a + b x)}{a (c + d x)} \right)^m - a b c x \left(\frac{c (a + b x)}{a (c + d x)} \right)^m (-c m + d (2+m) x) + \right. \\ & \quad \left. a^2 \left(d^2 x^2 - c^2 \left(-1 + \left(\frac{c (a + b x)}{a (c + d x)} \right)^m \right) - c d x \left(-2 + 2 \left(\frac{c (a + b x)}{a (c + d x)} \right)^m + m \left(\frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) \right) \Big) \Big) / \\ & \left(c (b c - a d)^2 (1+m) (2+m) \right) - \left(4 a c f h x^3 \text{AppellF1}[3, -m, 3+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \Big) / \\ & \left(-4 a c \text{AppellF1}[3, -m, 3+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - b c m x \text{AppellF1}[4, 1-m, 3+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \\ & \quad \left. a d (3+m) x \text{AppellF1}[4, -m, 4+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \frac{1}{d (2+m)} \\ & 3 e g \left(\frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[-2-m, -m, -1-m, \frac{b (c + d x)}{b c - a d}] \Big) \end{aligned}$$

Problem 132: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^3 (c + d x)^{-4-m} (e + f x)^m (g + h x) dx$$

Optimal (type 5, 815 leaves, 10 steps):

$$\begin{aligned} & \left((b c - a d)^2 (a d f + b (c f (2+m) - d e (3+m))) \right. \\ & \quad \left(c f h (4+m) - d (f g + e h (3+m)) \right) (c + d x)^{-3-m} (e + f x)^{1+m} \Big) / (d^4 f^2 (d e - c f) (3+m)) - \\ & \frac{1}{d^3 f^2} b (b c - a d) (c f h (4+m) - d (f g + e h (3+m))) (a + b x) (c + d x)^{-3-m} (e + f x)^{1+m} + \\ & \frac{h (a + b x)^3 (c + d x)^{-3-m} (e + f x)^{1+m}}{d f} - \\ & \left((b c - a d)^2 (3 a d f h - b (c f h (4+m) - d (f g + e h m))) \right) (c + d x)^{-2-m} (e + f x)^{1+m} \Big) / \\ & (d^4 f (d e - c f) (2+m)) + \\ & \left((b c - a d) (c f h (4+m) - d (f g + e h (3+m))) (2 a^2 d^2 f^2 + 2 a b d f (c f (1+m) - d e (3+m)) + \right. \\ & \quad \left. b^2 (c^2 f^2 (2+3 m+m^2) - 2 c d e f (3+4 m+m^2) + d^2 e^2 (6+5 m+m^2)) \right) \\ & (c + d x)^{-2-m} (e + f x)^{1+m} \Big) / (d^4 f^2 (d e - c f)^2 (2+m) (3+m)) - \\ & \left((b c - a d) (a d f - b (2 d e (2+m) - c f (3+2 m))) (3 a d f h - b (c f h (4+m) - d (f g + e h m))) \right) \\ & (c + d x)^{-1-m} (e + f x)^{1+m} \Big) / (d^4 f (d e - c f)^2 (1+m) (2+m)) - \\ & \left((b c - a d) (c f h (4+m) - d (f g + e h (3+m))) (2 a^2 d^2 f^2 + 2 a b d f (c f (1+m) - d e (3+m)) + \right. \\ & \quad \left. b^2 (c^2 f^2 (2+3 m+m^2) - 2 c d e f (3+4 m+m^2) + d^2 e^2 (6+5 m+m^2)) \right) (c + d x)^{-1-m} (e + f x)^{1+m} \Big) / \\ & (d^4 f (d e - c f)^3 (1+m) (2+m) (3+m)) - \frac{1}{d^5 f m} b^2 (3 a d f h - b (c f h (4+m) - d (f g + e h m))) \\ & (c + d x)^{-m} (e + f x)^m \left(\frac{d (e + f x)}{d e - c f} \right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, -\frac{f (c + d x)}{d e - c f}\right] \end{aligned}$$

Result (type 6, 10997 leaves):

$$\begin{aligned} & \left(3 a b^2 g (c + d x)^{-3-m} (e + f x)^m \right. \\ & \left(-2 d^3 e^3 x^3 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left(f (6 + 5 m + m^2) x + e \left(6 + 5 m + m^2 - 6 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) + \right. \\ & \quad c^3 \left(-2 e^2 f m x + e f^2 m (1+m) x^2 + f^3 (2+3 m+m^2) x^3 - 2 e^3 \left(-1 + \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) - \\ & \quad \left. 2 c^2 d e x \left(e f m (3+m) x + f^2 (3+4 m+m^2) x^2 + e^2 \left(-3 - m + 3 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) \Big) / \\ & \left(c (-d e + c f)^3 (1+m) (2+m) (3+m) \right) + \left(3 a^2 b h (c + d x)^{-3-m} (e + f x)^m \right. \\ & \left(-2 d^3 e^3 x^3 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left(f (6 + 5 m + m^2) x + e \left(6 + 5 m + m^2 - 6 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) + \right. \end{aligned}$$

$$\begin{aligned}
& c^3 \left(-2 e^2 f m x + e f^2 m (1+m) x^2 + f^3 (2+3m+m^2) x^3 - 2 e^3 \left(-1 + \left(\frac{e (c+d x)}{c (e+f x)} \right)^m \right) \right) - \\
& 2 c^2 d e x \left(e f m (3+m) x + f^2 (3+4m+m^2) x^2 + e^2 \left(-3-m + 3 \left(\frac{e (c+d x)}{c (e+f x)} \right)^m \right) \right) \Big/ \\
& \left(c (-d e + c f)^3 (1+m) (2+m) (3+m) \right) + \\
& \left(5 b^3 c e g x^4 (c+d x)^{-4-m} (e+f x)^m \text{AppellF1}[4, 4+m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] \right) \Big/ \\
& \left(4 \left(5 c e \text{AppellF1}[4, 4+m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] + c f m x \text{AppellF1}[5, 4+m, 1-m, \right. \right. \\
& \left. \left. 6, -\frac{d x}{c}, -\frac{f x}{e}] - d e (4+m) x \text{AppellF1}[5, 5+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}] \right) \right) + \\
& \left(15 a b^2 c e h x^4 (c+d x)^{-4-m} (e+f x)^m \text{AppellF1}[4, 4+m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] \right) \Big/ \\
& \left(4 \left(5 c e \text{AppellF1}[4, 4+m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] + c f m x \text{AppellF1}[5, 4+m, 1-m, \right. \right. \\
& \left. \left. 6, -\frac{d x}{c}, -\frac{f x}{e}] - d e (4+m) x \text{AppellF1}[5, 5+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}] \right) \right) + \\
& \left(6 b^3 c e h x^5 (c+d x)^{-4-m} (e+f x)^m \text{AppellF1}[5, 4+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}] \right) \Big/ \\
& \left(5 \left(6 c e \text{AppellF1}[5, 4+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}] + c f m x \text{AppellF1}[6, 4+m, 1-m, \right. \right. \\
& \left. \left. 7, -\frac{d x}{c}, -\frac{f x}{e}] - d e (4+m) x \text{AppellF1}[6, 5+m, -m, 7, -\frac{d x}{c}, -\frac{f x}{e}] \right) \right) + \\
& \left(3 a^2 b e g x^2 (c+d x)^{-3-m} \left(\frac{c+d x}{c} \right)^{4+m} \left(1 + \frac{d x}{c} \right)^{-4-m} (e+f x)^{-1+m} \left(\frac{e+f x}{e} \right)^{-m} \left(1 + \frac{f x}{e} \right)^{1+m} \right. \\
& \left. \left(c (4+m) (3 e + f x) \left(-2 d^3 e^3 x^3 + c^3 \left(-2 e^2 f m x \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + e f^2 m (1+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right. \right. \right. + \right. \\
& \left. \left. \left. \left. f^3 (2+3m+m^2) x^3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 2 e^3 \left(-1 + \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right) \right) - \right. \\
& \left. 2 c^2 d e x \left(e f m (3+m) x \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + f^2 (3+4m+m^2) x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m - e^2 (-3+ \right. \right. \\
& \left. \left. 3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right) + c d^2 e^2 x^2 \left(f (6+5m+m^2) x \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \\
& \left. \left. e \left(-6+6 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 5 m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + m^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right) \right) \text{Gamma}[4+m] - \\
& \left(2 d^4 e^4 (1+m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4+m) x) + c^4 \left(e^2 f^2 (-5+m) m x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \right. \\
& \left. \left. 2 e f^3 m (1+m) x^3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + f^4 (2+3m+m^2) x^4 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \right. \\
& \left. \left. 6 e^4 \left(-1 + \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) - 2 e^3 f x \left(4+m - 4 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 2 m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 2 c^3 d e x \left(2 e f^2 m (4 + m) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (4 + 5 m + m^2) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \\
& e^2 f (4 + m) x \left(3 - 3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) - e^3 \left(-8 + m + 8 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 2 \right. \\
& \left. m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) + c^2 d^2 e^2 x^2 \left(f^2 (12 + 7 m + m^2) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \\
& 2 e f (4 + m) x \left(-3 + 3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) + e^2 \left(m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \\
& \left. 12 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) + m \left(6 + 7 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \Gamma[5 + m] \Big) \Big) / \\
& \left(c \left(24 c^4 e^4 \Gamma[4 + m] + 6 c^4 e^4 m \Gamma[4 + m] + 96 c^3 d e^4 x \Gamma[4 + m] + \right. \right. \\
& 24 c^3 d e^4 m x \Gamma[4 + m] + 144 c^2 d^2 e^4 x^2 \Gamma[4 + m] + 36 c^2 d^2 e^4 m x^2 \Gamma[4 + m] + \\
& 96 c d^3 e^4 x^3 \Gamma[4 + m] + 24 c d^3 e^4 m x^3 \Gamma[4 + m] + 24 d^4 e^4 x^4 \Gamma[4 + m] + \\
& 6 d^4 e^4 m x^4 \Gamma[4 + m] - 24 c^4 e^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 6 c^4 e^4 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - 96 c^3 d e^4 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 48 c^3 d e^4 m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + 24 c^4 e^3 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 6 c^3 d e^4 m^2 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + 6 c^4 e^3 f m^2 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 144 c^2 d^2 e^4 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - 120 c^2 d^2 e^4 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + \\
& 96 c^3 d e^3 f m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - 12 c^4 e^2 f^2 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 33 c^2 d^2 e^4 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + 48 c^3 d e^3 f m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 15 c^4 e^2 f^2 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - 3 c^2 d^2 e^4 m^3 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + \\
& 6 c^3 d e^3 f m^3 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - 3 c^4 e^2 f^2 m^3 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 144 c d^3 e^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + 144 c^2 d^2 e^3 f x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 144 c^3 d e^2 f^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + 48 c^4 e f^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] - \\
& 228 c d^3 e^4 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] + 444 c^2 d^2 e^3 f m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \Gamma[4 + m] -
\end{aligned}$$

$$\begin{aligned}
& 348 c^3 d e^2 f^2 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 108 c^4 e f^3 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 132 c d^3 e^4 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 330 c^2 d^2 e^3 f m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 282 c^3 d e^2 f^2 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 84 c^4 e f^3 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 33 c d^3 e^4 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 93 c^2 d^2 e^3 f m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 87 c^3 d e^2 f^2 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 27 c^4 e f^3 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 3 c d^3 e^4 m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 9 c^2 d^2 e^3 f m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 9 c^3 d e^2 f^2 m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^4 e f^3 m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 96 c d^3 e^3 f x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 144 c^2 d^2 e^2 f^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 96 c^3 d e f^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 24 c^4 f^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 128 c d^3 e^3 f m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 264 c^2 d^2 e^2 f^2 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 192 c^3 d e f^3 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 50 c^4 f^4 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 62 c d^3 e^3 f m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 153 c^2 d^2 e^2 f^2 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 126 c^3 d e f^3 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 35 c^4 f^4 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 13 c d^3 e^3 f m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 36 c^2 d^2 e^2 f^2 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 33 c^3 d e f^3 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 10 c^4 f^4 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& c d^3 e^3 f m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^2 d^2 e^2 f^2 m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 3 c^3 d e f^3 m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + c^4 f^4 m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 6 c^4 e^4 \text{Gamma}[5 + m] - 24 c^3 d e^4 x \text{Gamma}[5 + m] - 36 c^2 d^2 e^4 x^2 \text{Gamma}[5 + m] - \\
& 24 c d^3 e^4 x^3 \text{Gamma}[5 + m] - 6 d^4 e^4 x^4 \text{Gamma}[5 + m] + \\
& 6 c^4 e^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c^3 d e^4 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] +
\end{aligned}$$

$$\begin{aligned}
& 6 c^3 d e^4 m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 e^3 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 36 c^2 d^2 e^4 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 21 c^2 d^2 e^4 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 24 c^3 d e^3 f m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 3 c^4 e^2 f^2 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^2 d^2 e^4 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^3 d e^3 f m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^4 e^2 f^2 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c d^3 e^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 26 c d^3 e^4 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 36 c^2 d^2 e^3 f m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 12 c^3 d e^2 f^2 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 2 c^4 e f^3 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 9 c d^3 e^4 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 21 c^2 d^2 e^3 f m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 15 c^3 d e^2 f^2 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^4 e f^3 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& c d^3 e^4 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^2 d^2 e^3 f m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^3 d e^2 f^2 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 e f^3 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 24 c d^3 e^3 f x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 36 c^2 d^2 e^2 f^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 24 c^3 d e f^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 26 c d^3 e^3 f m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 57 c^2 d^2 e^2 f^2 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 42 c^3 d e f^3 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 11 c^4 f^4 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 9 c d^3 e^3 f m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 24 c^2 d^2 e^2 f^2 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 21 c^3 d e f^3 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& c d^3 e^3 f m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^2 d^2 e^2 f^2 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^3 d e f^3 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 f^4 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] \Big) +
\end{aligned}$$

$$\begin{aligned}
& \left(a^3 e h x^2 (c + d x)^{-3-m} \left(\frac{c + d x}{c} \right)^{4+m} \left(1 + \frac{d x}{c} \right)^{-4-m} (e + f x)^{-1+m} \left(\frac{e + f x}{e} \right)^{-m} \left(1 + \frac{f x}{e} \right)^{1+m} \right. \\
& \left. \left(c (4+m) (3 e + f x) \left(-2 d^3 e^3 x^3 + c^3 \left(-2 e^2 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + e f^2 m (1 + m) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \right. \right. \\
& \left. \left. \left. \left. f^3 (2 + 3 m + m^2) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e^3 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) - \right. \\
& \left. 2 c^2 d e x \left(e f m (3 + m) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (3 + 4 m + m^2) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - e^2 (-3 + \right. \right. \\
& \left. \left. 3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) + c d^2 e^2 x^2 \left(f (6 + 5 m + m^2) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \\
& \left. e \left(-6 + 6 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 5 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \text{Gamma}[4 + m] - \\
& \left(2 d^4 e^4 (1 + m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4 + m) x) + c^4 \left(e^2 f^2 (-5 + m) m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \left. \left. 2 e f^3 m (1 + m) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^4 (2 + 3 m + m^2) x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \left. \left. 6 e^4 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) - 2 e^3 f x \left(4 + m - 4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 2 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) - \right. \\
& \left. 2 c^3 d e x \left(2 e f^2 m (4 + m) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (4 + 5 m + m^2) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \left. \left. e^2 f (4 + m) x \left(3 - 3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) - e^3 \left(-8 + m + 8 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 2 \right. \right. \right. \\
& \left. \left. \left. m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) + c^2 d^2 e^2 x^2 \left(f^2 (12 + 7 m + m^2) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \left. \left. 2 e f (4 + m) x \left(-3 + 3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) + e^2 \left(m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \right. \\
& \left. \left. \left. 12 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) + m \left(6 + 7 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \right) \text{Gamma}[5 + m] \right) \right) / \\
& \left(c \left(24 c^4 e^4 \text{Gamma}[4 + m] + 6 c^4 e^4 m \text{Gamma}[4 + m] + 96 c^3 d e^4 x \text{Gamma}[4 + m] + \right. \right. \\
& \left. \left. 24 c^3 d e^4 m x \text{Gamma}[4 + m] + 144 c^2 d^2 e^4 x^2 \text{Gamma}[4 + m] + 36 c^2 d^2 e^4 m x^2 \text{Gamma}[4 + m] + \right. \right. \\
& \left. \left. 96 c d^3 e^4 x^3 \text{Gamma}[4 + m] + 24 c d^3 e^4 m x^3 \text{Gamma}[4 + m] + 24 d^4 e^4 x^4 \text{Gamma}[4 + m] + \right. \right. \\
& \left. \left. 6 d^4 e^4 m x^4 \text{Gamma}[4 + m] - 24 c^4 e^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right. \\
& \left. \left. 6 c^4 e^4 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 96 c^3 d e^4 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right. \\
& \left. \left. 48 c^3 d e^4 m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 24 c^4 e^3 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right)
\end{aligned}$$

$$\begin{aligned}
& 6 c^3 d e^4 m^2 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 6 c^4 e^3 f m^2 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 144 c^2 d^2 e^4 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 120 c^2 d^2 e^4 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 96 c^3 d e^3 f m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 12 c^4 e^2 f^2 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 33 c^2 d^2 e^4 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 48 c^3 d e^3 f m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 15 c^4 e^2 f^2 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 3 c^2 d^2 e^4 m^3 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 6 c^3 d e^3 f m^3 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 3 c^4 e^2 f^2 m^3 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 144 c d^3 e^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 144 c^2 d^2 e^3 f x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 144 c^3 d e^2 f^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 48 c^4 e f^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 228 c d^3 e^4 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 444 c^2 d^2 e^3 f m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 348 c^3 d e^2 f^2 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 108 c^4 e f^3 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 132 c d^3 e^4 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 330 c^2 d^2 e^3 f m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 282 c^3 d e^2 f^2 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 84 c^4 e f^3 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 33 c d^3 e^4 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 93 c^2 d^2 e^3 f m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 87 c^3 d e^2 f^2 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 27 c^4 e f^3 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 3 c d^3 e^4 m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 9 c^2 d^2 e^3 f m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 9 c^3 d e^2 f^2 m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^4 e f^3 m^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 96 c d^3 e^3 f x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 144 c^2 d^2 e^2 f^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 96 c^3 d e f^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 24 c^4 f^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 128 c d^3 e^3 f m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 264 c^2 d^2 e^2 f^2 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] -
\end{aligned}$$

$$\begin{aligned}
& 192 c^3 d e f^3 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 50 c^4 f^4 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 62 c d^3 e^3 f m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 153 c^2 d^2 e^2 f^2 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 126 c^3 d e f^3 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 35 c^4 f^4 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 13 c d^3 e^3 f m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 36 c^2 d^2 e^2 f^2 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 33 c^3 d e f^3 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 10 c^4 f^4 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& c d^3 e^3 f m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^2 d^2 e^2 f^2 m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 3 c^3 d e f^3 m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + c^4 f^4 m^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 6 c^4 e^4 \text{Gamma}[5 + m] - 24 c^3 d e^4 x \text{Gamma}[5 + m] - 36 c^2 d^2 e^4 x^2 \text{Gamma}[5 + m] - \\
& 24 c d^3 e^4 x^3 \text{Gamma}[5 + m] - 6 d^4 e^4 x^4 \text{Gamma}[5 + m] + \\
& 6 c^4 e^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c^3 d e^4 x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 6 c^3 d e^4 m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 e^3 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 36 c^2 d^2 e^4 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 21 c^2 d^2 e^4 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 24 c^3 d e^3 f m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 3 c^4 e^2 f^2 m x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^2 d^2 e^4 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^3 d e^3 f m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^4 e^2 f^2 m^2 x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c d^3 e^4 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 26 c d^3 e^4 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 36 c^2 d^2 e^3 f m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 12 c^3 d e^2 f^2 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 2 c^4 e f^3 m x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 9 c d^3 e^4 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 21 c^2 d^2 e^3 f m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 15 c^3 d e^2 f^2 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^4 e f^3 m^2 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& c d^3 e^4 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^2 d^2 e^3 f m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] +
\end{aligned}$$

$$\begin{aligned}
& 3 c^3 d e^2 f^2 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 e f^3 m^3 x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 24 c d^3 e^3 f x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 36 c^2 d^2 e^2 f^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 24 c^3 d e f^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 26 c d^3 e^3 f m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 57 c^2 d^2 e^2 f^2 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 42 c^3 d e f^3 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 11 c^4 f^4 m x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 9 c d^3 e^3 f m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 24 c^2 d^2 e^2 f^2 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 21 c^3 d e f^3 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 m^2 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& c d^3 e^3 f m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^2 d^2 e^2 f^2 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^3 d e f^3 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 f^4 m^3 x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] \Big) + \\
& \left(a^3 f^3 g (e + f x)^{1+m} \left(c - \frac{d e}{f} + \frac{d (e + f x)}{f} \right)^{-m} \left(1 + \frac{d (e + f x)}{\left(c - \frac{d e}{f} \right) f} \right)^m \right. \\
& \left. \text{Hypergeometric2F1}\left[1 + m, 4 + m, 2 + m, -\frac{d (e + f x)}{\left(c - \frac{d e}{f} \right) f}\right] \right) / \left((-d e + c f)^4 (1 + m) \right)
\end{aligned}$$

Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^2 (c + d x)^{-4-m} (e + f x)^m (g + h x) dx$$

Optimal (type 5, 572 leaves, 9 steps):

$$\begin{aligned}
& \left((b c - a d) (d g - c h) (a d f + b (c f (2 + m) - d e (3 + m))) (c + d x)^{-3-m} (e + f x)^{1+m} \right) / \\
& \quad \left(d^3 f (d e - c f) (3 + m) \right) - \frac{b (d g - c h) (a + b x) (c + d x)^{-3-m} (e + f x)^{1+m}}{d^2 f} - \\
& \quad \frac{(b c - a d)^2 h (c + d x)^{-2-m} (e + f x)^{1+m}}{d^3 (d e - c f) (2 + m)} - \left((d g - c h) (b^2 (d e - c f) (2 + m) (c f (1 + m) - d e (3 + m))) - \right. \\
& \quad \left. 2 d f (b^2 c e + a^2 d f + a b (c f (1 + m) - d e (3 + m))) \right) / \\
& \quad \left(c + d x \right)^{-2-m} (e + f x)^{1+m} / \left(d^3 f (d e - c f)^2 (2 + m) (3 + m) \right) - \\
& \quad \left((b c - a d) h (a d f - b (2 d e (2 + m) - c f (3 + 2 m))) (c + d x)^{-1-m} (e + f x)^{1+m} \right) / \\
& \quad \left(d^3 (d e - c f)^2 (1 + m) (2 + m) \right) + \\
& \quad \left((d g - c h) (b^2 (d e - c f) (2 + m) (c f (1 + m) - d e (3 + m))) - \right. \\
& \quad \left. 2 d f (b^2 c e + a^2 d f + a b (c f (1 + m) - d e (3 + m))) \right) (c + d x)^{-1-m} (e + f x)^{1+m} / \\
& \quad \left(d^3 (d e - c f)^3 (1 + m) (2 + m) (3 + m) \right) - \frac{1}{d^4 m} b^2 h (c + d x)^{-m} (e + f x)^m \left(\frac{d (e + f x)}{d e - c f} \right)^{-m} \\
& \text{Hypergeometric2F1}\left[-m, -m, 1 - m, -\frac{f (c + d x)}{d e - c f}\right]
\end{aligned}$$

Result (type 6, 5412 leaves):

$$\begin{aligned}
& \left(b^2 g (c + d x)^{-3-m} (e + f x)^m \right. \\
& \quad \left(-2 d^3 e^3 x^3 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left(f (6 + 5 m + m^2) x + e \left(6 + 5 m + m^2 - 6 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) + \right. \\
& \quad c^3 \left(-2 e^2 f m x + e f^2 m (1 + m) x^2 + f^3 (2 + 3 m + m^2) x^3 - 2 e^3 \left(-1 + \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) - \\
& \quad \left. 2 c^2 d e x \left(e f m (3 + m) x + f^2 (3 + 4 m + m^2) x^2 + e^2 \left(-3 - m + 3 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) / \\
& \quad \left(c (-d e + c f)^3 (1 + m) (2 + m) (3 + m) \right) + \left(2 a b h (c + d x)^{-3-m} (e + f x)^m \right. \\
& \quad \left(-2 d^3 e^3 x^3 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left(f (6 + 5 m + m^2) x + e \left(6 + 5 m + m^2 - 6 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) + \right. \\
& \quad c^3 \left(-2 e^2 f m x + e f^2 m (1 + m) x^2 + f^3 (2 + 3 m + m^2) x^3 - 2 e^3 \left(-1 + \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) - \\
& \quad \left. 2 c^2 d e x \left(e f m (3 + m) x + f^2 (3 + 4 m + m^2) x^2 + e^2 \left(-3 - m + 3 \left(\frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) / \\
& \quad \left(c (-d e + c f)^3 (1 + m) (2 + m) (3 + m) \right) + \\
& \quad \left(5 b^2 c e h x^4 (c + d x)^{-4-m} (e + f x)^m \text{AppellF1}\left[4, 4 + m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}\right] \right) / \\
& \quad \left(4 \left(5 c e \text{AppellF1}\left[4, 4 + m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}\right] + \right. \right. \\
& \quad \left. \left. c f m x \text{AppellF1}\left[5, 4 + m, 1 - m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& d e (4+m) \times \text{AppellF1}\left[5, 5+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right] \Bigg) + \left(2 a b g x^2 (c+d x)^{-3-m} (e+f x)^m \right. \\
& \left. \left(c (4+m) (3 e+f x) \left(-2 d^3 e^3 x^3 + c^3 \left(-2 e^2 f m x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + e f^2 m (1+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right.\right. \right. \right. \\
& \left. \left. \left. \left. f^3 (2+3 m+m^2) x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 e^3 \left(-1+\left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right)\right) \right) - \right. \\
& \left. 2 c^2 d e x \left(e f m (3+m) x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^2 (3+4 m+m^2) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m - e^2 (-3+ \right. \right. \\
& \left. \left. 3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) \right) + c d^2 e^2 x^2 \left(f (6+5 m+m^2) x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \\
& \left. \left. e \left(-6+6 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 5 m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right)\right) \right) \text{Gamma}[4+m] - \right. \\
& \left. \left(2 d^4 e^4 (1+m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4+m) x) + c^4 \left(e^2 f^2 (-5+m) m x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \right. \right. \\
& \left. \left. \left. 2 e f^3 m (1+m) x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^4 (2+3 m+m^2) x^4 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \right. \right. \\
& \left. \left. \left. 6 e^4 \left(-1+\left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) - 2 e^3 f x \left(4+m-4 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right)\right) - \right. \\
& \left. 2 c^3 d e x \left(2 e f^2 m (4+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^3 (4+5 m+m^2) x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \right. \\
& \left. \left. e^2 f (4+m) x \left(3-3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) - e^3 \left(-8+m+8 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 \right. \right. \right. \\
& \left. \left. \left. m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) \right) + c^2 d^2 e^2 x^2 \left(f^2 (12+7 m+m^2) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \\
& \left. 2 e f (4+m) x \left(-3+3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) + e^2 \left(m^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \right. \\
& \left. \left. 12 \left(-1+\left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) + m \left(6+7 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right)\right) \right) \right) \text{Gamma}[5+m] \Bigg) \Bigg) / \\
& \left(c \left((4+m) \left(6 d^4 e^4 x^4 + c^4 \left(6 e^3 f m x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m - 3 e^2 f^2 m (1+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. 3 e f^3 (1+m) (2+m)^2 x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^4 (6+11 m+6 m^2+m^3) x^4 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m - \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. 6 e^4 \left(-1+\left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) - 3 c^3 d e x \left(-2 e^2 f m (4+m) x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. e f^2 (12+26 m+17 m^2+3 m^3) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^3 (8+14 m+7 m^2+m^3) \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 e^3 \left(-4+4 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right)\right) \right) - \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& c d^3 e^3 x^3 \left(f (24 + 26 m + 9 m^2 + m^3) \times \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 3 e (-8 + 12 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \\
& \quad \left. 16 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 7 m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m) \right) + 3 c^2 d^2 e^2 x^2 \\
& \left(e f (12 + 34 m + 19 m^2 + 3 m^3) \times \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (12 + 19 m + 8 m^2 + m^3) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \\
& \quad \left. e^2 \left(7 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \text{Gamma}[4 + m] + \\
& \left(-6 d^4 e^4 x^4 + c^4 \left(-6 e^3 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 3 e^2 f^2 m (1 + m) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \right. \\
& \quad \left. \left. e f^3 m (2 + 3 m + m^2) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - f^4 (6 + 11 m + 6 m^2 + m^3) x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \quad \left. \left. 6 e^4 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) + 3 c^3 d e x \left(-2 e^2 f m (4 + m) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \quad \left. \left. e f^2 m (4 + 5 m + m^2) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (8 + 14 m + 7 m^2 + m^3) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \quad \left. \left. 2 e^3 \left(-4 + 4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) - 3 c^2 d^2 e^2 x^2 \right. \\
& \quad \left(e f m (12 + 7 m + m^2) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (12 + 19 m + 8 m^2 + m^3) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \\
& \quad \left. e^2 \left(7 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) + \\
& c d^3 e^3 x^3 \left(f (24 + 26 m + 9 m^2 + m^3) \times \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + e \left(26 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 9 m^2 \right. \right. \\
& \quad \left. \left. \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 24 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \text{Gamma}[5 + m] \right) + \\
& \left(a^2 h x^2 (c + d x)^{-3-m} (e + f x)^m \left(c (4 + m) (3 e + f x) \left(-2 d^3 e^3 x^3 + \right. \right. \right. \\
& \quad \left. \left. \left. c^3 \left(-2 e^2 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + e f^2 m (1 + m) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \right. \\
& \quad \left. \left. \left. f^3 (2 + 3 m + m^2) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e^3 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) - \right. \\
& \quad \left. 2 c^2 d e x \left(e f m (3 + m) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (3 + 4 m + m^2) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \right. \\
& \quad \left. \left. e^2 \left(-3 + 3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) + c d^2 e^2 x^2 \left(f (6 + 5 m + m^2) x \right. \right. \\
& \quad \left. \left. \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + e \left(-6 + 6 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 5 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Gamma}[4+m] - \left(2 d^4 e^4 (1+m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4+m) x) + \right. \\
& c^4 \left(e^2 f^2 (-5+m) m x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 2 e f^3 m (1+m) x^3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \\
& f^4 (2+3 m+m^2) x^4 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 6 e^4 \left(-1 + \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) - \\
& 2 e^3 f x \left(4+m - 4 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 2 m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \Big) - \\
& 2 c^3 d e x \left(2 e f^2 m (4+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + f^3 (4+5 m+m^2) x^3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \\
& e^2 f (4+m) x \left(3 - 3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) - \\
& e^3 \left(-8+m+8 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 2 m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) + \\
& c^2 d^2 e^2 x^2 \left(f^2 (12+7 m+m^2) x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 2 e f (4+m) x \right. \\
& \left(-3+3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) + e^2 \left(m^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 12 \right. \\
& \left. \left. \left. \left(-1 + \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) + m \left(6+7 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right) \right) \right) \text{Gamma}[5+m] \Big) / \\
& \left(c \left((4+m) \left(6 d^4 e^4 x^4 + c^4 \left(6 e^3 f m x \left(\frac{c (e+f x)}{e (c+d x)} \right)^m - 3 e^2 f^2 m (1+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \right. \right. \right. \right. \\
& 3 e f^3 (1+m) (2+m)^2 x^3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + f^4 (6+11 m+6 m^2+m^3) x^4 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m - \\
& \left. \left. \left. \left. \left. \left. 6 e^4 \left(-1 + \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) - 3 c^3 d e x \left(-2 e^2 f m (4+m) x \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \right. \right. \right. \right. \\
& e f^2 (12+26 m+17 m^2+3 m^3) x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + f^3 (8+14 m+7 m^2+m^3) \\
& x^3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 2 e^3 \left(-4+4 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \Big) - \\
& c d^3 e^3 x^3 \left(f (24+26 m+9 m^2+m^3) x \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 3 e \left(-8+12 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + \right. \right. \\
& \left. \left. 16 m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 7 m^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + m^3 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right) + 3 c^2 d^2 e^2 x^2 \\
& \left(e f (12+34 m+19 m^2+3 m^3) x \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + f^2 (12+19 m+8 m^2+m^3) x^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m - \right. \\
& \left. e^2 \left(7 m \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + m^2 \left(\frac{c (e+f x)}{e (c+d x)} \right)^m + 12 \left(-1 + \left(\frac{c (e+f x)}{e (c+d x)} \right)^m \right) \right) \right) \right) \text{Gamma}[4+m] +
\end{aligned}$$

$$\begin{aligned}
& \left(-6 d^4 e^4 x^4 + c^4 \left(-6 e^3 f m x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 3 e^2 f^2 m (1 + m) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \right. \\
& \quad \left. \left. e f^3 m (2 + 3 m + m^2) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - f^4 (6 + 11 m + 6 m^2 + m^3) x^4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \quad \left. \left. 6 e^4 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) + 3 c^3 d e x \left(-2 e^2 f m (4 + m) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \quad \left. \left. e f^2 m (4 + 5 m + m^2) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (8 + 14 m + 7 m^2 + m^3) x^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \quad \left. \left. 2 e^3 \left(-4 + 4 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) - 3 c^2 d^2 e^2 x^2 \right. \\
& \quad \left(e f m (12 + 7 m + m^2) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (12 + 19 m + 8 m^2 + m^3) x^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \\
& \quad \left. e^2 \left(7 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) + \\
& \quad c d^3 e^3 x^3 \left(f (24 + 26 m + 9 m^2 + m^3) x \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + e \left(26 m \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 9 m^2 \right. \right. \\
& \quad \left. \left. \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + m^3 \left(\frac{c (e + f x)}{e (c + d x)} \right)^m + 24 \left(-1 + \left(\frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \Gamma [5 + m] \right) + \\
& \left(a^2 f^3 g (e + f x)^{1+m} \left(c - \frac{d e}{f} + \frac{d (e + f x)}{f} \right)^{-m} \left(1 + \frac{d (e + f x)}{\left(c - \frac{d e}{f} \right) f} \right)^m \right. \\
& \quad \left. \text{Hypergeometric2F1} \left[\begin{array}{l} 1 + \\ m, 4 + \\ m, 2 + m, \\ - \frac{d (e + f x)}{\left(c - \frac{d e}{f} \right) f} \end{array} \right] \right) / \left((-d e + c f)^4 (1 + m) \right)
\end{aligned}$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int \frac{(A + B x) (c + d x)^n (e + f x)^p}{a + b x} dx$$

Optimal (type 6, 177 leaves, 5 steps):

$$-\left(\left((A b - a B) (c + d x)^{1+n} (e + f x)^p \left(\frac{d (e + f x)}{d e - c f} \right)^{-p} \text{AppellF1}[1 + n, 1, -p, 2 + n, \frac{b (c + d x)}{b c - a d}, -\frac{f (c + d x)}{d e - c f}] \right) / (b (b c - a d) (1 + n)) \right) -$$

$$\left(B (c + d x)^{1+n} (e + f x)^{1+p} \text{Hypergeometric2F1}[1, 2 + n + p, 2 + p, \frac{d (e + f x)}{d e - c f}] \right) / (b (d e - c f) (1 + p))$$

Result (type 6, 692 leaves):

$$\begin{aligned} & \frac{1}{b^2 f} (c + d x)^n (e + f x)^p \\ & \left(\left(A b d f^2 (-1 + n + p) (a + b x) \text{AppellF1}[-n - p, -n, -p, 1 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] \right) / \right. \\ & \quad \left((n + p) \right. \\ & \quad \left(d f (-1 + n + p) (a + b x) \text{AppellF1}[-n - p, -n, -p, 1 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] + \right. \\ & \quad \left(-b c + a d \right) f n \text{AppellF1}[1 - n - p, 1 - n, -p, 2 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] + \\ & \quad \left. d (-b e + a f) p \text{AppellF1}[1 - n - p, -n, 1 - p, 2 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] \right) \left. \right) - \\ & \left(a B d f^2 (-1 + n + p) (a + b x) \text{AppellF1}[-n - p, -n, -p, 1 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] \right) / \\ & \quad \left((n + p) \right. \\ & \quad \left(d f (-1 + n + p) (a + b x) \text{AppellF1}[-n - p, -n, -p, 1 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] + \right. \\ & \quad \left(-b c + a d \right) f n \text{AppellF1}[1 - n - p, 1 - n, -p, 2 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] + \\ & \quad \left. d (-b e + a f) p \text{AppellF1}[1 - n - p, -n, 1 - p, 2 - n - p, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}] \right) \left. \right) + \\ & \frac{1}{1 + p} b B \left(\frac{f (c + d x)}{-d e + c f} \right)^{-n} (e + f x) \text{Hypergeometric2F1}[-n, 1 + p, 2 + p, \frac{d (e + f x)}{d e - c f}] \end{aligned}$$

Problem 137: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (A + B x) (c + d x)^{-m}}{e + f x} dx$$

Optimal (type 5, 233 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{d (B e - A f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^2 m} - \frac{1}{f^2 m} \\
 & \quad (B e - A f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}] - \\
 & \quad \left((a B d f m - b (B d e - A d f + B c f m)) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \right. \\
 & \quad \left. \text{Hypergeometric2F1}[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}] \right) / (b (b c - a d) f^2 m (1 + m))
 \end{aligned}$$

Result (type 6, 627 leaves):

$$\begin{aligned}
 & \left((a + b x)^m (c + d x)^{-m} \right. \\
 & \quad \left(-B d (-b c + a d) e (b e - a f) (-1 + m) (2 + m) (a + b x) \text{AppellF1}[1 + m, m, 1, 2 + m, \right. \\
 & \quad \left. \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + A d (-b c + a d) f (b e - a f) (-1 + m) (2 + m) (a + b x) \right. \\
 & \quad \left. \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + b B (1 + m) \left(\frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \right. \\
 & \quad \left. (e + f x) \left((b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \right. \\
 & \quad \left. (a + b x) \left((-b c f + a d f) \text{AppellF1}[2 + m, m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \right. \\
 & \quad \left. \left. d (-b e + a f) m \text{AppellF1}[2 + m, 1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \\
 & \quad \left. \text{Hypergeometric2F1}[1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d}] \right) / \left(b d f (1 - m) (1 + m) (e + f x) \right. \\
 & \quad \left. \left((b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \right. \\
 & \quad \left. (a + b x) \left((-b c f + a d f) \text{AppellF1}[2 + m, m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \right. \\
 & \quad \left. \left. d (-b e + a f) m \text{AppellF1}[2 + m, 1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right)
 \end{aligned}$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \frac{(A + B x) (c + d x)^n (e + f x)^p}{\sqrt{a + b x}} dx$$

Optimal (type 6, 250 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{b^2} 2 (A b - a B) \sqrt{a + b x} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \\ & \text{AppellF1} \left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right] + \frac{1}{3 b^2} 2 B (a + b x)^{3/2} (c + d x)^n \\ & \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} \left[\frac{3}{2}, -n, -p, \frac{5}{2}, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right] \end{aligned}$$

Result (type 6, 551 leaves):

$$\begin{aligned} & \frac{1}{3 b^2} 2 (b c - a d) (b e - a f) \sqrt{a + b x} (c + d x)^n (e + f x)^p \\ & \left(\left(9 (A b - a B) \text{AppellF1} \left[\frac{1}{2}, -n, -p, \frac{3}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \right. \\ & \left(3 (b c - a d) (b e - a f) \text{AppellF1} \left[\frac{1}{2}, -n, -p, \frac{3}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] - \right. \\ & 2 (a + b x) \left(d (-b e + a f) n \text{AppellF1} \left[\frac{3}{2}, 1-n, -p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + \right. \\ & \left. \left. (-b c + a d) f p \text{AppellF1} \left[\frac{3}{2}, -n, 1-p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) + \right. \\ & \left(5 B (a + b x) \text{AppellF1} \left[\frac{3}{2}, -n, -p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \\ & \left(5 (b c - a d) (b e - a f) \text{AppellF1} \left[\frac{3}{2}, -n, -p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] - \right. \\ & 2 (a + b x) \left(d (-b e + a f) n \text{AppellF1} \left[\frac{5}{2}, 1-n, -p, \frac{7}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + \right. \\ & \left. \left. (-b c + a d) f p \text{AppellF1} \left[\frac{5}{2}, -n, 1-p, \frac{7}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) \right) \end{aligned}$$

Problem 139: Unable to integrate problem.

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^3 dx$$

Optimal (type 6, 530 leaves, 31 steps):

$$\begin{aligned}
& \frac{1}{b^4 (1+m)} (b g - a h)^3 (a + b x)^{1+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \\
& \quad \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [1+m, -n, -p, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^4 (2+m)} 3 h (b g - a h)^2 (a + b x)^{2+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \\
& \quad \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [2+m, -n, -p, 3+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^4 (3+m)} 3 h^2 (b g - a h) (a + b x)^{3+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \\
& \quad \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [3+m, -n, -p, 4+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^4 (4+m)} h^3 (a + b x)^{4+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \\
& \quad \text{AppellF1} [4+m, -n, -p, 5+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}]
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^3 dx$$

Problem 140: Unable to integrate problem.

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^2 dx$$

Optimal (type 6, 393 leaves, 15 steps):

$$\begin{aligned}
& \frac{1}{b^3 (1+m)} (b g - a h)^2 (a + b x)^{1+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \\
& \quad \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [1+m, -n, -p, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^3 (2+m)} 2 h (b g - a h) (a + b x)^{2+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \\
& \quad \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [2+m, -n, -p, 3+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^3 (3+m)} h^2 (a + b x)^{3+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left(\frac{b (e + f x)}{b e - a f} \right)^{-p} \\
& \quad \text{AppellF1} [3+m, -n, -p, 4+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}]
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^2 dx$$

Problem 141: Unable to integrate problem.

$$\int (a+b x)^m (c+d x)^n (e+f x)^p (g+h x) dx$$

Optimal (type 6, 256 leaves, 7 steps) :

$$\begin{aligned} & \frac{1}{b^2 (1+m)} (b g - a h) (a+b x)^{1+m} (c+d x)^n \left(\frac{b (c+d x)}{b c - a d} \right)^{-n} (e+f x)^p \\ & \left(\frac{b (e+f x)}{b e - a f} \right)^{-p} \text{AppellF1} [1+m, -n, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] + \\ & \frac{1}{b^2 (2+m)} h (a+b x)^{2+m} (c+d x)^n \left(\frac{b (c+d x)}{b c - a d} \right)^{-n} (e+f x)^p \left(\frac{b (e+f x)}{b e - a f} \right)^{-p} \\ & \text{AppellF1} [2+m, -n, -p, 3+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] \end{aligned}$$

Result (type 8, 29 leaves) :

$$\int (a+b x)^m (c+d x)^n (e+f x)^p (g+h x) dx$$

Problem 142: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^m (c+d x)^n (e+f x)^p dx$$

Optimal (type 6, 123 leaves, 3 steps) :

$$\begin{aligned} & \frac{1}{b (1+m)} (a+b x)^{1+m} (c+d x)^n \left(\frac{b (c+d x)}{b c - a d} \right)^{-n} (e+f x)^p \\ & \left(\frac{b (e+f x)}{b e - a f} \right)^{-p} \text{AppellF1} [1+m, -n, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] \end{aligned}$$

Result (type 6, 296 leaves) :

$$\begin{aligned} & \left((b c - a d) (b e - a f) (2+m) (a+b x)^{1+m} (c+d x)^n \right. \\ & \left. (e+f x)^p \text{AppellF1} [1+m, -n, -p, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) / \\ & \left(b (1+m) \left((b c - a d) (b e - a f) (2+m) \text{AppellF1} [1+m, -n, -p, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) - \right. \\ & (a+b x) \left(d (-b e + a f) n \text{AppellF1} [2+m, 1-n, -p, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] + \right. \\ & \left. \left. (-b c + a d) f p \text{AppellF1} [2+m, -n, 1-p, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) \right) \end{aligned}$$

Problem 144: Unable to integrate problem.

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-m-n} dx$$

Optimal (type 6, 268 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{b^2 (1+m)} (A b - a B) (a + b x)^{1+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \\ & \left(\frac{b (e + f x)}{b e - a f} \right)^{m+n} \text{AppellF1}[1+m, -n, m+n, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\ & \frac{1}{b^2 (2+m)} B (a + b x)^{2+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \left(\frac{b (e + f x)}{b e - a f} \right)^{m+n} \\ & \text{AppellF1}[2+m, -n, m+n, 3+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] \end{aligned}$$

Result (type 8, 35 leaves):

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-m-n} dx$$

Problem 145: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-1-m-n} dx$$

Optimal (type 6, 283 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{b f (1+m)} B (a + b x)^{1+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \\ & \left(\frac{b (e + f x)}{b e - a f} \right)^{m+n} \text{AppellF1}[1+m, -n, m+n, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] - \\ & \left((B e - A f) (a + b x)^{1+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \left(\frac{b (e + f x)}{b e - a f} \right)^{m+n} \right. \\ & \left. \text{AppellF1}[1+m, -n, 1+m+n, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] \right) / (f (b e - a f) (1+m)) \end{aligned}$$

Result (type 6, 576 leaves):

$$\begin{aligned}
& \frac{1}{b(1+m)} (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^n \\
& (e + f x)^{-m-n} \left(\left(B \text{AppellF1}[1+m, -n, m+n, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) \right. \\
& \left(f \left((b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1+m, -n, m+n, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) - \right. \\
& (a + b x) \left(d (-b e + a f) n \text{AppellF1}[2+m, 1-n, m+n, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) + \\
& (b c - a d) f (m+n) \text{AppellF1}[2+m, -n, 1+m+n, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \left. \right) \right) + \\
& \left(\left(A - \frac{B e}{f} \right) \text{AppellF1}[1+m, -n, 1+m+n, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) \right) / \\
& \left((e + f x) \left((b c - a d) (b e - a f) (2 + m) \right. \right. \\
& \left. \left. \text{AppellF1}[1+m, -n, 1+m+n, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) - (a + b x) \right. \\
& \left. \left(d (-b e + a f) n \text{AppellF1}[2+m, 1-n, 1+m+n, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) + (b c - a \right. \\
& \left. d) f (1+m+n) \text{AppellF1}[2+m, -n, 2+m+n, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) \right) \right)
\end{aligned}$$

Problem 147: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-3-m-n} dx$$

Optimal (type 5, 263 leaves, 3 steps):

$$\begin{aligned}
& \frac{(B e - A f) (a + b x)^{1+m} (c + d x)^{1+n} (e + f x)^{-2-m-n}}{(b e - a f) (d e - c f) (2 + m + n)} - \\
& \left((b (B c e (1 + m) + A (c f (1 + n) - d e (2 + m + n)))) + \right. \\
& a (A d f (1 + m) + B (d e (1 + n) - c f (2 + m + n)))) \\
& (a + b x)^{1+m} (c + d x)^n \left(\frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{-n} (e + f x)^{-1-m-n} \\
& \left. \text{Hypergeometric2F1}[1+m, -n, 2+m, -\frac{(d e - c f) (a + b x)}{(b c - a d) (e + f x)}] \right) / \\
& ((b e - a f)^2 (d e - c f) (1 + m) (2 + m + n))
\end{aligned}$$

Result (type 5, 10558 leaves):

$$\left(A (a + b x)^{1+2m} (c + d x)^{2n} \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-6-2m-2n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \right)$$

$$\begin{aligned}
& \left(1 - \frac{f(a+b x)}{-b e + a f}\right)^{-2-m-n} \Gamma[2+m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \left. \frac{f(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \right. \\
& \left. \left((d e - c f) (a+b x) \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \right. \\
& \left. \left((b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n] \right) - \right. \\
& \left. \left(f(-d e + c f) (a+b x)^2 \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \right. \right. \\
& \left. \left. \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \left((b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[-n] \right) \right) / \\
& \left(b (1+m) \left(-\frac{1}{(-b e + a f) (1+m)} f(-2-m-n) (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} \right. \right. \\
& \left. \left. (e + f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-3-m-n} \right. \right. \\
& \left. \left. \Gamma[2+m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \right. \right. \\
& \left. \left. \left. m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \Gamma[3+m] \right. \right. \\
& \left. \left. \left(f(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / ((b e - a f) \right. \right. \\
& \left. \left. \Gamma[3+m]) + \left((d e - c f) (a+b x) \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \right. \right. \\
& \left. \left. m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / ((b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n]) - \right. \\
& \left. \left(f(-d e + c f) (a+b x)^2 \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \right. \right. \\
& \left. \left. \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / ((b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[-n]) \right) - \\
& \frac{1}{(-b c + a d) (1+m)} d n (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \\
& \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^{-1+n} \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n}
\end{aligned}$$

$$\begin{aligned}
& \text{Gamma}[2+m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \quad \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \\
& \quad \left. \left(f (a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad ((b e - a f) \text{Gamma}[3+m]) + \left((d e - c f) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad ((b e - a f) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]) - \\
& \quad \left. \left(f (-d e + c f) (a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) - \\
& \quad \frac{1}{(-b e + a f) (1+m)} f (3+m+n) (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \\
& \quad \left(\frac{-b e - b f x}{-b e + a f} \right)^{2+m+n} \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \\
& \quad \text{Gamma}[2+m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \quad \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \\
& \quad \left. \left(f (a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad ((b e - a f) \text{Gamma}[3+m]) + \left((d e - c f) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad ((b e - a f) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]) - \\
& \quad \left. \left(f (-d e + c f) (a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) + \\
& \quad \frac{1}{b (1+m)} f (-3-m-n) (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-4-m-n} \\
& \quad \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n}
\end{aligned}$$

$$\begin{aligned}
& \text{Gamma}[2+m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \quad \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \\
& \quad \left. \left(f (a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f) \text{Gamma}[3+m] \right) + \left((d e - c f) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad \left((b e - a f) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) - \\
& \quad \left. \left(f (-d e + c f) (a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) + \\
& \quad \frac{1}{(-b c + a d) (1+m)} d n (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-1-n} (e+f x)^{-3-m-n} \\
& \quad \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \\
& \quad \text{Gamma}[2+m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \quad \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\text{Gamma}[3+m]} + \\
& \quad \left. \left(f (a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f) \text{Gamma}[3+m] \right) + \left((d e - c f) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad \left((b e - a f) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) - \\
& \quad \left. \left(f (-d e + c f) (a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) + \\
& \quad \frac{1}{b (1+m)} d n (a+b x)^{1+m} (c+d x)^{-1+n} \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \\
& \quad \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2+m]
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{2 \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\operatorname{Gamma}[3+m]} + \right. \\
& \quad \left. \frac{m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\operatorname{Gamma}[3+m]} + \right. \\
& \quad \left. \left(f (a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f) \operatorname{Gamma}[3+m] \right) + \left((d e - c f) (a+b x) \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad \left((b e - a f) (c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n] \right) - \\
& \quad \left(f (-d e + c f) (a+b x)^2 \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad \left((b e - a f)^2 (c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n] \right) + \\
& \quad \left((a+b x)^m (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \right. \\
& \quad \left. \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \operatorname{Gamma}[2+m] \right) / \\
& \quad \left(2 \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right. \\
& \quad \left. \right) / \\
& \quad \left(\frac{m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\operatorname{Gamma}[3+m]} + \right. \\
& \quad \left. \left(f (a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \right) / \\
& \quad \left((b e - a f) \operatorname{Gamma}[3+m] \right) + \left((d e - c f) (a+b x) \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad \left((b e - a f) (c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n] \right) - \\
& \quad \left(f (-d e + c f) (a+b x)^2 \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) / \\
& \quad \left((b e - a f)^2 (c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n] \right) + \\
& \quad \left((d e - c f) (a+b x) \right) / \\
& \quad \left(\frac{1}{b (1+m)} (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \right. \\
& \quad \left. \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \operatorname{Gamma}[2+m] \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{b f \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) \text{Gamma}[3 + m]} - \\
& \left(2 n \left(-\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)}\right) \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((3 + m) \text{Gamma}[3 + m]) - \\
& \left(m n \left(-\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)}\right) \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((3 + m) \text{Gamma}[3 + m]) - \\
& \left(f n (a + b x) \left(-\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)}\right) \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((b e - a f) (3 + m) \text{Gamma}[3 + m]) - \\
& \left(d (d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((b e - a f) (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[-n]) + \\
& \left(d f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((b e - a f)^2 (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[-n]) + \\
& \left(b (d e - c f) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]) - \\
& \left(2 b f (-d e + c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]) + \\
& \left(2 (d e - c f) (1 - n) (a + b x) \left(-\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)}\right) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]\right) / ((b e - a f) (4 + m) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]) - \\
& \left(2 f (-d e + c f) (1 - n) (a + b x)^2 \left(-\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)}\right)\right)
\end{aligned}$$

$$\begin{aligned}
 & \left. \left(\left(b e - a f \right)^2 (4 + m) (c + d x) \Gamma[4 + m] \Gamma[-n] \right) \right\} - \\
 & \left(b e (a + b x)^{1+2m} (c + d x)^{2n} \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-6-2m-2n} \right. \\
 & \quad \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \\
 & \quad \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \\
 & \quad \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
 & \quad \Gamma[\\
 & \quad 2 + \\
 & \quad m] \\
 & \quad \left. \frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\Gamma[3 + m]} + \right. \\
 & \quad \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\Gamma[3 + m]} + \right. \\
 & \quad \left. \frac{f (a + b x) \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \Gamma[3 + m]} + \right. \\
 & \quad \left((d e - c f) (a + b x) \Gamma[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \right. \\
 & \quad \left((b e - a f) (c + d x) \Gamma[4 + m] \Gamma[-n] \right) - \\
 & \quad \left. \left(f (-d e + c f) (a + b x)^2 \Gamma[1 - n] \right. \right. \\
 & \quad \left. \left. \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \right\} / \\
 & \quad \left((b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n] \right) \right\} / \\
 & \left(b f (1 + m) \left(-\frac{1}{(-b e + a f) (1 + m)} f (-2 - m - n) (a + b x)^{1+m} (c + d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} \right. \right. \\
 & \quad \left. \left. (e + f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-3-m-n} \right) \right. \\
 & \quad \left. \left. \Gamma[2 + m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\Gamma[3 + m]} + \right. \right. \right. \\
 & \quad \left. \left. \left. \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \frac{m \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \\
& \left(f (a + b x) \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / ((b e - a f) \\
& \quad \text{Gamma}[3 + m]) + \left((d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + \right. \\
& \quad \left. m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / ((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]) - \\
& \left(f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \right. \\
& \quad \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / ((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]) - \\
& \frac{1}{(-b c + a d) (1 + m)} d n (a + b x)^{1+m} (c + d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \\
& \quad \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^{-1+n} \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
& \text{Gamma}[2 + m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \right. \\
& \quad \left. m \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \\
& \quad \text{Gamma}[3 + m] \\
& \left(f (a + b x) \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \\
& \quad ((b e - a f) \text{Gamma}[3 + m]) + \left((d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - \right. \\
& \quad \left. n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / ((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]) - \\
& \left(f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \right. \\
& \quad \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / ((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]) - \\
& \frac{1}{(-b e + a f) (1 + m)} f (3 + m + n) (a + b x)^{1+m} (c + d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \\
& \quad \left(\frac{-b e - b f x}{-b e + a f} \right)^{2+m+n} \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
& \text{Gamma}[2 + m] \left(\frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \\
& \left(f (a + b x) \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f) \text{Gamma}[3 + m] \right) + \left((d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
& \left(f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) + \\
& \frac{1}{b (1 + m)} f (-3 - m - n) (a + b x)^{1+m} (c + d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-4-m-n} \\
& \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
& \text{Gamma}[2 + m] \left(\frac{2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \right. \\
& \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \\
& \left. \left(f (a + b x) \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \right. \\
& \left((b e - a f) \text{Gamma}[3 + m] \right) + \left((d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
& \left(f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) + \\
& \frac{1}{(-b c + a d) (1 + m)} d n (a + b x)^{1+m} (c + d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-1-n} (e + f x)^{-3-m-n} \\
& \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
& \text{Gamma}[2 + m] \left(\frac{2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \\
& \left(f (a + b x) \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f) \text{Gamma}[3 + m] \right) + \left((d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
& \left(f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) + \\
& \frac{1}{b (1 + m)} d n (a + b x)^{1+m} (c + d x)^{-1+n} \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \\
& \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2 + m] \\
& \left(2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \text{Gamma}[3 + m] + \\
& \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \\
& \left(f (a + b x) \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f) \text{Gamma}[3 + m] \right) + \left((d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
& \left(f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \left((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) + \\
& (a + b x)^m (c + d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \\
& \left(1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2 + m] \\
& \left(2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \text{Gamma}[3 + m]
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{m Hypergeometric2F1}\left[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{\text{Gamma}[3+m]} + \\
& \left(f (a+b x) \text{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / \\
& \left((b e - a f) \text{Gamma}[3+m] \right) + \left((d e - c f) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}\left[2, \right. \right. \\
& \left. \left. 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / \left((b e - a f) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) - \\
& \left(f (-d e + c f) (a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, \right. \right. \\
& \left. \left. \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / \left((b e - a f)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) + \\
& \frac{1}{b (1+m)} (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \\
& \left(1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left(1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2+m] \\
& \left(\frac{b f \text{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{(b e - a f) \text{Gamma}[3+m]} - \right. \\
& \left. \left(2 n \left(-\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \right. \right. \\
& \left. \left. \text{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / ((3+m) \text{Gamma}[3+m]) - \right. \\
& \left. \left(m n \left(-\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \text{Hypergeometric2F1}\left[\right. \right. \right. \\
& \left. \left. \left. 2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / ((3+m) \text{Gamma}[3+m]) - \right. \\
& \left. \left(f n (a+b x) \left(-\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \text{Hypergeometric2F1}\left[\right. \right. \right. \\
& \left. \left. \left. 2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / ((b e - a f) (3+m) \text{Gamma}[3+m]) - \right. \\
& \left. \left(d (d e - c f) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, \right. \right. \right. \\
& \left. \left. \left. \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / ((b e - a f) (c+d x)^2 \text{Gamma}[4+m] \text{Gamma}[-n]) + \right. \\
& \left. \left(d f (-d e + c f) (a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, \right. \right. \right. \\
& \left. \left. \left. \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) / ((b e - a f) (c+d x)^2 \text{Gamma}[4+m] \text{Gamma}[-n])
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)} \right] \Bigg) \Bigg/ \left((b e - a f)^2 (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[-n] \right) + \\
& \left(b (d e - c f) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \Bigg/ \\
& \left((b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
& \left(2 b f (-d e + c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \right. \\
& \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \Bigg/ \left((b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) + \\
& \left(2 (d e - c f) (1 - n) (a + b x) \left(-\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \right. \\
& \left. \text{Gamma}[1 - n] \text{Hypergeometric2F1}[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \Bigg/ \\
& \left((b e - a f) (4 + m) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
& \left(2 f (-d e + c f) (1 - n) (a + b x)^2 \left(-\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \right. \\
& \left. \text{Gamma}[1 - n] \text{Hypergeometric2F1}[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \Bigg/ \\
& \left. \left((b e - a f)^2 (4 + m) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) \right) \Bigg) + \\
& \left(B (a + b x)^{1+m} (c + d x)^n \left(\frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{-n} \right. \\
& \left. (e + f x)^{-1-m-n} \text{Hypergeometric2F1}[\right. \\
& \left. 1 + \right. \\
& \left. m, -n, 2 + \right. \\
& \left. m, \right. \\
& \left. \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)}] \Bigg) \Bigg/ (f \\
& (b e - a f) (1 + \\
& m))
\end{aligned}$$

Problem 148: Attempted integration timed out after 120 seconds.

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-4-m-n} dx$$

Optimal (type 5, 558 leaves, 4 steps):

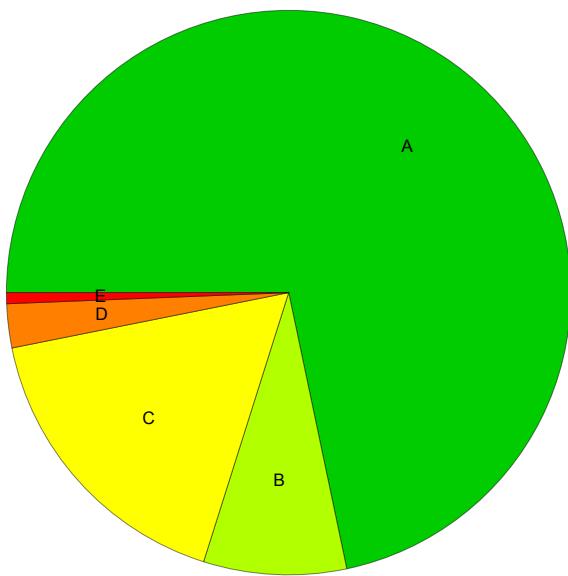
$$\begin{aligned}
& \frac{\left(B e - A f\right) \left(a + b x\right)^{1+m} \left(c + d x\right)^{1+n} \left(e + f x\right)^{-3-m-n}}{\left(b e - a f\right) \left(d e - c f\right) \left(3 + m + n\right)} + \\
& \left(\left(a f \left(A d f \left(2 + m\right) + B \left(d e \left(1 + n\right) - c f \left(3 + m + n\right)\right)\right)\right) + \right. \\
& \quad b \left(B e \left(d e + c f \left(1 + m\right)\right) + A f \left(c f \left(2 + n\right) - d e \left(4 + m + n\right)\right)\right) \left(a + b x\right)^{1+m} \\
& \quad \left.\left(c + d x\right)^{1+n} \left(e + f x\right)^{-2-m-n}\right) / \left(\left(b e - a f\right)^2 \left(d e - c f\right)^2 \left(2 + m + n\right) \left(3 + m + n\right)\right) + \\
& \quad \frac{1}{\left(b e - a f\right)^3 \left(d e - c f\right)^2 \left(1 + m\right) \left(2 + m + n\right) \left(3 + m + n\right)} \\
& \quad \left(\left(2 + m + n\right) \left(a b c d f \left(B e - A f\right) + b d e \left(\left(a B c f + A \left(b d e - b c f - a d f\right)\right) \left(3 + m + n\right) - \right.\right.\right. \\
& \quad \left.\left.\left.\left(B e - A f\right) \left(b c \left(1 + m\right) + a d \left(1 + n\right)\right)\right) - \left(b c + a d\right) f\right. \right. \\
& \quad \left.\left.\left(\left(a B c f + A \left(b d e - b c f - a d f\right)\right) \left(3 + m + n\right) - \left(B e - A f\right) \left(b c \left(1 + m\right) + a d \left(1 + n\right)\right)\right)\right) - \right. \\
& \quad \left.\left.\left(b c \left(1 + m\right) + a d \left(1 + n\right)\right) \left(a f \left(A d f \left(2 + m\right) + B \left(d e \left(1 + n\right) - c f \left(3 + m + n\right)\right)\right)\right) + \right. \right. \\
& \quad \left.b \left(B e \left(d e + c f \left(1 + m\right)\right) + A f \left(c f \left(2 + n\right) - d e \left(4 + m + n\right)\right)\right)\right) \\
& \quad \left(a + b x\right)^{1+m} \left(c + d x\right)^n \left(\frac{\left(b e - a f\right) \left(c + d x\right)}{\left(b c - a d\right) \left(e + f x\right)}\right)^{-n} \left(e + f x\right)^{-1-m-n} \\
& \text{Hypergeometric2F1}\left[1 + m, -n, 2 + m, -\frac{\left(d e - c f\right) \left(a + b x\right)}{\left(b c - a d\right) \left(e + f x\right)}\right]
\end{aligned}$$

Result (type 1, 1 leaves):

???

Summary of Integration Test Results

159 integration problems



A - 114 optimal antiderivatives

B - 13 more than twice size of optimal antiderivatives

C - 27 unnecessarily complex antiderivatives

D - 4 unable to integrate problems

E - 1 integration timeouts